Key Engineering Materials Vols 274-276 (2004) pp 35-42 *Online: 2004-10-15*
© (2004) Trans Tech Publications, Switzerland
doi:10.4028/www.scientific.net/KEM.274-276.35

Micromechanics of nonlinear composites

Gengkai Hu[1,a] and Zhuping Huang[2,b]

[1] Department of Applied Mechanics, Beijing Institute of Technology, 100081, Beijing, China

[2] Department of Mechanics and Engineering Science, Peking University, 100871, Beijing, China

[a]email: hugeng@public.bta.net.cn, [b]email: huangzp@pku.edu.cn

Keywords: micropolar composites, Micromechanics, plasticity, size effect, second order moment.

Abstract. Two homogenization methods are presented for modeling the overall elasto-plastic behavior of composite materials, the first one is applied to the case where the nonlocal property of the constituents can be neglected, and the second one considers this nonlocal effect by idealizing the constituents as micropolar materials. The secant moduli method and Ponte Castañeda's variational method are discussed in these two homogenization methods.

Introduction

There are different length scales involved in the micromechanical modeling for heterogeneous materials: the structural characteristic size L, the size of representative volume element (RVE) l, the characteristic length scale of the inhomogeneity A (for example the particle size, the inter-distance of the particles) and the characteristic length scale of the matrix l_m (grain size of a polycrystalline material for example). For different length scale conditions, different homogenization techniques should be employed. For example, the classical micromechanical method applies for the following length scale condition $L >> l >> A >> l_m$ [1,2], so that both the constituents and the homogenized materials can be idealized as Cauchy materials without any inner microstructures. However with the emerge of new technologies such as nanotechology, MEMS, thin films, the structure size L is usually small and the length scale condition necessary for the classical micromechanics is subjected to an acute scrutiny; on the other hand, with the innovation of new composite materials, such as metal matrix composites (MMCs), nanocomposites, foam composites, the size of the filler usually has the same order as the characteristic length scale (l_m) of the matrix. For these length scale conditions, different homogenization strategies have to be proposed [2,3]: for thin films and MEMS, usually $L \approx l$, a high order continuum model should be assigned for the homogenized macro-element; and for MMCs, nanocomposites, more often $A \approx l_m$, a high order continuum model may be used to describe properly the nonlocal response of the matrix material. The different material models and the corresponding length scale conditions are summarized in table 1.

Table 1 material models and length scale conditions

Constituent model		length scale condition	homogenized model
Cauchy model	$A >> l_m$	$L >> l >> A$	Cauchy model
High order continuum	$A \approx l_m$		
Cauchy model	$A >> l_m$	$L \approx l \approx A$	High order continuum
High order continuum	$A \approx l_m$		

In this paper, a brief review on the advances of homogenization methods for nonlinear composites will be presented. Two length scale conditions, $L >> l >> A >> l_m$ and $L >> l >> A \approx l_m$ will be discussed separately.

Homogenization of nonlinear composites under the condition $L >> l >> A >> l_m$

As discussed in the introduction, under this length scale condition both the constituents and the homogenized materials can be idealized as Cauchy materials, the progress on the nonlinear composites along this line can be found in the review paper given by Ponte Castañeda and Suquet[4]. In the following, we will discuss the secant moduli method based on the second order stress moment [5,6], which is shown to be equivalent to the method proposed by Ponte Castañeda[7]. The method chooses a series of linear comparison composites (characterized by the effective stress potential $\overline{W}^s$ or the effective compliance tensor $\overline{\boldsymbol{M}}^s$) to describe the nonlinear behavior of the actual composite, these linear comparison composites have the same microstructure as that in the composite to be analyzed, however the elastic compliance $\boldsymbol{M}_r^s$ of the phase *r* of the linear comparison composite is set to have the secant compliance of the nonlinear phase *r* (characterized by the stress potential w_r) of the actual composite, this means

$$\boldsymbol{M}_r^s : s = \frac{\partial w_r}{\partial s}. \tag{1}$$

The stress potential w_r of the nonlinear phase *r* may have different particular forms according to specific materials, for an isotropic Von Mises material with a power type hardening law, it is

$$w_r = w_r^0 + \frac{1}{18K_r}\overline{s}^2. \tag{2}$$

where $\overline{s} = s_{ii}$, and for $\overline{s}_e > s_{yr}$

$$w_r^0 = \frac{\overline{s}_e^2}{6m_r} + \frac{n_r}{n_r+1}\frac{1}{H_r^{1/n_r}}\left(\overline{s}_e - s_{yr}\right)^{\frac{n_r+1}{n_r}}. \tag{3}$$

where m_r, K_r are the elastic shear and bulk moduli of the phase *r*, s_{yr} and n_r, H_r are respectively the initial yield stress, and hardening parameters of the phase *r*, $\overline{s}_e = \sqrt{3s'_{ij}s'_{ij}/2}$, and s'_{ij} is the deviatoric stress. From equations (2,3), the secant shear and bulk moduli of the phase *r* are defined by

$$m_r^s = \frac{1}{1/m_r + 3[(\overline{s}_e - s_{yr})/H_r]^{1/n_r}/\overline{s}_e}, \quad K_r^s = K_r. \tag{4}$$

For the phase *r* in the actual composite, the plastic strain is usually not uniform, so its secant moduli are in fact position-dependant. In order to describe this heterogeneous plastic strain more precisely, a more refined approach can be proposed by dividing the phase *r* into some sub-regions, and in each of such region the secant moduli are assumed to be constant, this usually makes the problem difficult to be solved analytically. So, for simplicity, we utilize the homogenized secant moduli for each phase by taking $\overline{s}^2 = < s^2 >_r$, $\overline{s}_e^2 = 3 < s'_{ij}s'_{ij} >_r /2$ in equations (3,4), where $< \bullet >_r$ denotes the volume average of the said quantity over the phase *r*. For the linear comparison composite with the *r*th linear phase characterized by $\boldsymbol{M}_r^s$, its effective compliance $\overline{\boldsymbol{M}}^s$ can be evaluated analytically by many micromechanical methods[8]. Taking the Mori-Tanaka's method for example, for a two-phase composite with aligned elastic ellipsoidal inclusions, the effective compliance tensor of the linear comparison composite is given by

$$\overline{\boldsymbol{M}}^s = \boldsymbol{M}_0^s + c_1\left[(\boldsymbol{M}_1 : (\boldsymbol{M}_0^s)^{-1} - \boldsymbol{I})^{-1} + (1-c_1)(\boldsymbol{I} - \boldsymbol{S})\right]^{-1} : \boldsymbol{M}_0^s. \tag{5}$$

where c_1 is the volume fraction of the inclusion, $\boldsymbol{I}$ is fourth order identity tensor, $\boldsymbol{S}$ is Eshelby tensor, indices 0,1 are referred to quantities related to the matrix and the inclusion respectively. For composites with spherical inclusions, the effective shear and bulk moduli of the linear comparison composite can be written as

$$\frac{\overline{m}^s}{m_0^s} = 1 + \frac{c_1}{(1-c_1)x_1 + m_0^s/(m_1 - m_0^s)}, \quad \frac{\overline{K}^s}{K_0} = 1 + \frac{c_1}{(1-c_1)x_2 + K_0/(K_1 - K_0)}. \tag{6}$$

where

$$x_1 = \frac{6(K_0 + 2m_0^s)}{5(3K_0 + 4m_0^s)}, \quad x_2 = \frac{3K_0}{3K_0 + 4m_0^s}.$$

For a given applied load S, if the corresponding secant shear modulus of the nonlinear phase (here the matrix) m_0^s is assumed to be known, then the effective compliance tensor $\overline{\boldsymbol{M}}^s$ of the linear comparison composite can be evaluated by equations (5 or 6), which is considered as the secant effective compliance of the nonlinear composite at the applied load S, and the composite strain is simply calculated by

$$\boldsymbol{E} = \overline{\boldsymbol{M}}^s : S. \tag{7}$$

To determine the variation of m_0^s as a function of the applied load S, we consider again the linear comparison composite under the applied load S, the micro-macro transition principle leads to

$$< s : \boldsymbol{M}^s : s > = S : \overline{\boldsymbol{M}}^s : S. \tag{8}$$

Under a constant applied load S with a small variation $d\boldsymbol{M}^s$, it can be shown that

$$< s : d\boldsymbol{M}^s : s > = S : d\overline{\boldsymbol{M}}^s : S. \tag{9}$$

If we only consider a variation of the shear modulus of the matrix dm_0^s, then

$$\overline{s}_e^2 = \frac{3}{2} < s'_{ij} s'_{ij} >_0 = \frac{1}{1-c_1} S : \left(-\frac{3 {m_0^s}^2 \partial \overline{\boldsymbol{M}}^s}{\partial m_0^s}\right) : S. \tag{10}$$

This equation can be used to determine the corresponding secant shear modulus m_0^s of the matrix for a given load S, the stress and strain relation of the nonlinear composite can then be constructed according to the secant moduli method. This version of the secant moduli method can also be recast into a beautiful variational expression proposed by Ponte Castañeda [7] for a composite with N nonlinear phases:

$$\overline{W}_{eff} \geq \overline{W}^-_{eff} \equiv \sup_{\forall \boldsymbol{M}^s} \left[\overline{W}^s(\boldsymbol{M}^s, S) - \sum_{r=1}^{N} c_r \sup_{\forall s} \left[w_r^s(s) - w_r(s) \right] \right]. \tag{11}$$

where c_r is the volume fraction of the phase r, $\overline{W}_{eff}$ is the stress potential of the nonlinear composite. The first optimization procedure imposes that the constituents of the linear comparison material have the secant moduli of the corresponding nonlinear phase, and the second optimization procedure determines the variation of the secant moduli of the individual phase as a function of the applied load (equation 10).

This method has been widely used to determine the effective behavior of nonlinear composites, the most advantage of the method is that the results obtained for linear composites can be utilized, however this method fails to describe the particle size-dependence in predicting the nonlinear behavior of the composite, widely observed in metal matrix composites [9], and this will be discussed in the next section.

Homogenization of nonlinear composites under the condition $L >> l >> A \approx l_m$

When the particle size is comparable to the intrinsic length of the matrix material, a high order continuum model has to be assigned for the matrix material, discussions on the high order theory can be found in references [10,11]. Wei[12], Chen and Wang[13] have used high order continuum models to describe the size-dependence of the overall behavior for composite materials. Here in this paper, the matrix material will be idealized as a micropolar material.

Micropolar elasticity and plasticity

In the micropolar theory, three rotation angles f_i are introduced in additional to the macro-displacement u_i of a material point. The kinematic, equilibrium and constitutive equations for an isotropic micropolar material are (body force and couple are neglected)

$$\begin{cases} e_{ij} = u_{j,i} - e_{kij} f_k \\ k_{ij} = f_{j,i} \end{cases}, \quad \begin{cases} s_{ij,j} = 0 \\ m_{ij,j} + e_{jik} s_{ik} = 0 \end{cases}, \quad \begin{cases} s_{ij} = C_{ijkl} e_{kl} + B_{ijkl} k_{kl} \\ m_{ij} = B_{ijkl} e_{kl} + D_{ijkl} k_{kl} \end{cases}. \quad (12)$$

It should be noted that the stress and the strain are not symmetric in micropolar theory. m_{ij}, k_{ij} are the couple stress and the torsion respectivley. For a centrosymmetric and isotropic micropolar material, $B_{ijkl} = 0$, and

$$C_{ijkl} = l d_{ij} d_{kl} + (m+k) d_{jk} d_{il} + (m-k) d_{ik} d_{jl}, \; D_{ijkl} = a d_{ij} d_{kl} + (b+g) d_{jk} d_{il} + (b-g) d_{ik} d_{jl}. \quad (13)$$

where (m, l) are the classical Lame constants, (k, g, b, a) are new material constants introduced in micropolar theory, and d_{ij} is the Kronecker delta. With $s'_{(ij)}, s_{<ij>}, s \; (\equiv s_{ii})$ and $e'_{(ij)}, e_{<ij>}, e \; (\equiv e_{ii})$ denoting separately the deviatoric symmetric, anti-symmetric and hydrostatic parts of the stress and the strain tensors, and similar notations for the couple-stress and torsion tensors, the well-established elastic constitutive relations for a linear isotropic micropolar material can be rewritten as [14]:

$$s'_{(ij)} = 2m e'_{(ij)}, \; s'_{<ij>} = 2k e'_{<ij>}, \; s = 3Ke; \; m'_{(ij)} = 2b k'_{(ij)}, \; m'_{<ij>} = 2g e'_{<ij>}, \; m = 3Pk. \quad (14)$$

where $K = l + 2/3m$ is the bulk modulus, $P = a + 2/3b$ can be regarded as the corresponding stiffness measure for the torsion, and symbols ( ) and < > in the subscript denote the symmetric and anti-symmetric parts of a second order tensor, respectively.

There are two distinct sets of moduli: (m, l, k) which relate the traditional stresses and strains and have the dimension of force per unit area, and (g, b, a) which relate the higher-order couple-stresses and torsions, with the dimension of force. Due to the dimensional difference between these two sets of moduli, at least three intrinsic characteristic lengths can be defined for an isotropic elastic micropolar material. These elastic micropolar length parameters can be defined in different ways; in this paper, they are defined by [16]:

$$l_1 = (g/m)^{1/2}, \qquad l_2 = (b/m)^{1/2}, \qquad l_3 = (a/m)^{1/2}. \quad (15)$$

For simplicity, in the following discussion, we let $l_1 = l_2 = l_3 = l_m$.

For a nonlinear micropolar material, the generalized equivalent stress may be defined as[11,15,16]:

$$\bar{s}_e = \sqrt{\frac{3}{2}s'_{(ij)}s'_{(ij)} + \frac{3}{2}l_m^{-2}(m'_{(ij)}m'_{(ij)} + m_{<ij>}m_{<ij>})}\ . \tag{16}$$

The stress potential for a nonlinear micropolar material may then be written as [16]:

$$w = w_0(\bar{s}_e) + \frac{1}{6k}s^2_{<e>} + \frac{1}{18K}s^2 + \frac{1}{18P}m^2 . \tag{17}$$

$w_0(\bar{s}_e)$ has the same form as in equation (3) with a new interpretation for the equivalent stress, and $s^2_{<e>} = 3s_{<ij>}s_{<ij>}/2$.

For the deformation theory in plasticity, the secant moduli of a nonlinear micropolar material at the stress state $\bar{s}_e$ can be defined from equation(17)

$$m^s = \frac{1}{1/m + 3[(\bar{s}_e - s_y)/H]^{1/n}/\bar{s}_e},\ \ k^s = k\ ,\ K^s = K\ ;\ b^s = l_m^2 m^s,\ g^s = l_m^2 m^s,\ P^s = P . \tag{18}$$

Micro-macro transition method and effective classical moduli of the composite

At the length scale condition $L >> l >> A \approx l_m$, the size of the RVE is sufficiently small that it can still be considered as a material point in the structure, the classical boundary condition is applied on the RVE: $u_i = E_{(ij)}x_j$, $f_i = 0$ or $F_i = S_{(ij)}n_j$, $m_{ij}n_j = 0$, where n_i is the unit normal of the boundary of the RVE. Under the above boundary conditions, it can be shown that

$$< s : e + \boldsymbol{m} : \boldsymbol{k} >= \boldsymbol{E}^{sym} : \overline{\boldsymbol{C}}^{sym} : \boldsymbol{E}^{sym} = S^{sym} : \overline{\boldsymbol{M}}^{sym} : S^{sym} . \tag{19}$$

Equation (19) in fact defines the classical stiffness or compliance tensor $\overline{\boldsymbol{C}}^{sym}$ or $\overline{\boldsymbol{M}}^{sym}$ of the micropolar composite, which relates the symmetrical stress and strain. To determine the effective stiffness tensor $\overline{\boldsymbol{C}}^{sym}$, it is necessary to evaluate the local stress and couple stress in the RVE, and then the relation between $< s_{(ij)} >$ and $< e_{(ij)} >= E_{(ij)}$ (under linear displacement boundary condition) provides the classical stiffness tensor of the micropolar composite, or alternatively the relation between $< e_{(ij)} >$ and $< s_{(ij)} >= S_{(ij)}$ (under uniform traction boundary condition) gives the effective compliance. To compute the average strain and torsion for different phases, the average equivalent inclusion method will be used, a detailed explanation may be found in references[16,17]. For a two-phase particulate composite, the effective shear and bulk moduli of the composite estimated by Mori-Tanaka's method can also be expressed by equation (6), but the constants x_1, x_2 should be replaced by

$$x_1 = \frac{6(K_0 + 2m_0^s)}{5(3K_0 + 4m_0^s)} - \frac{6k_0}{5(k_0 + m_0^s)}G(h),\ x_2 = \frac{3K_0}{3K_0 + 4m_0^s} . \tag{20}$$

where $G(h) = e^{-h}(h^{-2} + h^{-3})[h\cosh h - \sinh h]$, $h = a/h$, $h^2 = (m_0^s + k_0)(g_0^s + b_0^s)/4m_0^s k_0$, and a denotes the radius of the particle. Here in equation (20), the superscript s is used for the secant notation for the matrix material, which will be used as a linear comparison micropolar composite in the following discussion.

It can be verified that when $a/h \to \infty$, the classical micromechanical results can be recovered, as expected. However when the particle size is small, the effective shear modulus is larger than that in the classical prediction, and the effective bulk modulus will not depend on the particle size.

Secant moduli method for micropolar composites

In order to determine the overall nonlinear property of a micropolar composite, the same approach as in the classical micromechanics discussed previously may be employed. The key point is to evaluate the secant moduli of the nonlinear micropolar matrix as a function of the applied load S^{sym}. To this end, we consider a linear comparison micropolar composite $\overline{\boldsymbol{M}}_s^{sym}$ characterized locally by the compliance tensors $\boldsymbol{M}^s, \boldsymbol{R}^s$. Suppose that the local material compliances have the variations $d\boldsymbol{M}^s$, $d\boldsymbol{R}^s$, these will lead to the variations of the local stress, couple stress and the macroscopic compliance of the composite $\boldsymbol{ds}$, $\boldsymbol{dm}$ and $d\overline{\boldsymbol{M}}_s^{sym}$. Then from equation (19), we have [16,17]

$$< s : d\boldsymbol{M}^s : s + \boldsymbol{m} : d\boldsymbol{R}^s : \boldsymbol{m} >= S^{sym} : d\overline{\boldsymbol{M}}_s^{sym} : S^{sym}. \tag{21}$$

Now only the following variations dm_0^s, db_0^s, dg_0^s of the matrix are considered, then from equation (21), the average equivalent stress of the matrix in the linear comparison micropolar composite defined by equation (16) can be obtained

$$\overline{s}_e = \sqrt{\frac{1}{1-c_1} S^{sym} : \left[-3m_0^{s\,2} \left(\frac{\partial \overline{\boldsymbol{M}}_s^{sym}}{\partial m_0^s} + \frac{\partial \overline{\boldsymbol{M}}_s^{sym}}{\partial b_0^s} + \frac{\partial \overline{\boldsymbol{M}}_s^{sym}}{\partial g_0^s} \right) \right] : S^{sym}} . \tag{22}$$

With the aid of the effective moduli given by equations (6, 20), the yield function of a micropolar composite can then be estimated from equation (22) by setting $\overline{s}_e = s_y$ (s_y is the initial yield stress of the matrix). When the matrix material has undergone a plastic deformation, the secant moduli method can be utilized to determine the overall nonlinear stress and strain relation of the composite, the method is the same as explained in the classical micromechanics.

The capacity of the present model can be illustrated by the following examples, only particulate composites are considered. The influence of the mechanical property and the size of the particle on the initial yield surface is shown in Fig1 1 ($S_{eff} = \sqrt{3S'_{(ij)}S'_{(ij)}/2}$, $S = S_{ii}$). In the computation, $k_0/m_0 = 0.5, K_0/m_0 = 2.53$, $c_1 = 30\%$, and for the common particles $m_1/m_0 = K_1/K_0 = 10$.

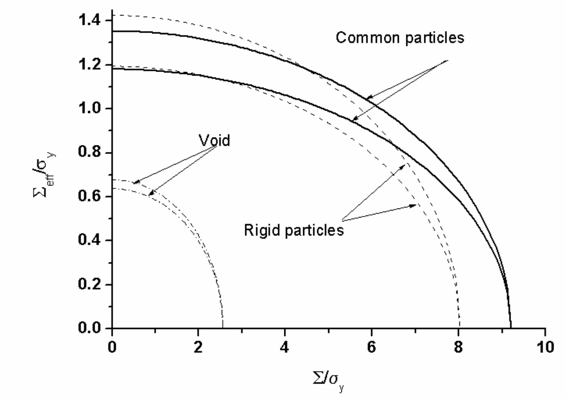

Fig. 1 initial yield surface of composites

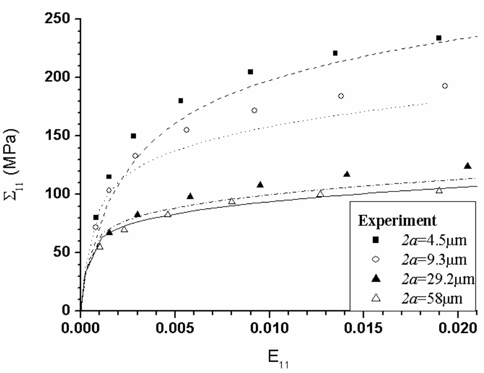

Fig. 2 comparison between theory and experiment

Two particle sizes are examined $a = l_m$, $a = 100 l_m$, corresponding the outer and inner lines for each classs of the particle in figure 1. For $a = 100 l_m$, the proposed theory is in fact reduced to the classical micromechanical method, however if the particle'size is comparable to the matrix characteristic length l_m, a size-dependent initial yield stress is observed. This size-dependence is more pronounced for a pure shear loading and for hard particles. The comparison between the modeling and the experimental results conducted by Kouzeli and Mortensen[9] is shown in Fig. 2 for different paricle sizes and volume concentrations, the following material constants are used in the modeling: $K_1/K_0 = 3.71$, $m_1/m_0 = 6.52$, $K_0/m_0 = 2.61$, $k_0/m_0 = 0.5$ and $l_m = 4.2mm$. Four sets of composite samples are examined, with the particle diameter $2a$ and the volume fraction c_1 separately given by: (4.5mm, 0.39), (9.3mm, 0.54), (29.2mm, 0.461) and (58mm, 0.475). An excellent correlation exists between the modeling and the experimental observation

Similar to the case in the classical micromechanics, the proposed secant moduli method based on second order stress and couple stress moments can also have a variational interpretation of the Ponte Castañeda's type, which can be written in the following form.

$$\overline{W}_{eff} \geq \overline{W}^{-}_{eff} \equiv \sup_{\forall \boldsymbol{M}^s, \boldsymbol{R}^s} \left[\overline{W}^s(\boldsymbol{M}^s, \boldsymbol{R}^s, S^{sym}) - \sum_{r=1}^{N} c_r \sup_{\forall s, \boldsymbol{m}} \left[w_r^s(s, \boldsymbol{m}) - w_r(s, \boldsymbol{m}) \right] \right]. \quad (23)$$

where w_r is the stress potential of the nonlinear phase *r*, and w_r^s is the stress potential for a linear micropolar material associated with the phase *r*. c_r is the volume fraction of the phase *r*. $\overline{W}^s$ is the effective stress potential of a linear comparison composite, characterized locally by stress potentials w_r^s. $\overline{W}_{eff}$ is the stress potential of the nonlinear composite.

Discussion and Summary

There are, of cause, different approaches to include the size effect in a micromechanical formulation: firstly different high order material models can be utilized for the constituent materials, for example strain gradient theory[12] or gradient plasticity[18]; or a more radically different approach which incorporates the interfacial energy. The main problem encountered in the high order theory is the determination and interpretation of the high order material constants, more fundamental works are still needed to clarify these points.

To summarize, we have discussed some homogenization methods applied for two different length scale conditions, the first one is for the case where the particle size is very large compared to the characteristic length of the matrix, the second one is that the microstructure of the matrix material cannot be neglected. The nonlocal nature of the coarse-grain structure of the matrix is modeled by a micropolar model, secant moduli method and Ponte Castañeda's variational method are extended to micropolar composites, particle size effects are then well captured analytically.

Acknowledgement: the work is supported by NSFC through grants10032010, 10332020 and 10372004.

References

[1] A. Zaoui. Journal of Engineering Mechanics-ASCE Vol. 128 (2002), p.808

[2] G.K. Hu, X.Liu and F. Xun, Advance in Mechanics (in press, in Chinese, 2004)

[3] S. Forest, K. Sab, Mech. Res. Comm. Vol.25 (1998), p.449

[4] Ponte Castañeda, P. Suquet, Advance in Applied Mechanics, 1998, 171-302

[5] P. Suquet, C. R. Acad. Sci. Paris Ser. II320 (1995), p.563

[6] G.K.Hu, Int. J. Plasticity, Vol.12 (1996), p.439

[7] P. Ponte Castañeda, J. Mech. Phys. Solids Vol.39 (1991), p.45

[8] S.Nemat-Nasser, M. Hori, *Micromechanics: Overall Properties of Heterogeneous Materials* (North-Holland: Elsevier, 1993).
[9] M. Kouzeli, A. Mortensen, Acta Materialia Vol.50 (2002), p.39
[10] A.C. Eringen, *Microcontinuum, field theory* (Springer 1999)
[11] N.A Fleck, J.W. Hutchinson, Advances in Applied Mechanics Vol.33 (1997), p.295
[12] Y.G. Wei, Acta Mech. Sinica. 17 (2001), p.45
[13] S.H. Chen, T. Wang, Acta Mechanica Vol.157 (2002), p.113
[14] W. Nowacki, *Theory of asymmetric elasticity* (Pergamon press 1986)
[15] H. Lippmann, ASME Appl. Mech. Rev. Vol. 48(1995), p753
[16] X.N.Liu,G.K. Hu,Int. J. of Plasticity (in press 2004)
[17] F. Xun, G.K. Hu, Z.P. Huang, Int. J. Solids and Structures Vol.41 (2004), p.247
[18] I. Tsagrakis, E.C. Aifantis, J. of Engineering Materials and Technology Vol.124 (2002.), p.352

Advances in Engineering Plasticity and Its Applications
10.4028/www.scientific.net/KEM.274-276

Micromechanics of Nonlinear Composites
10.4028/www.scientific.net/KEM.274-276.35

DOI References

[1] A. Zaoui. Journal of Engineering Mechanics-ASCE Vol. 128 (2002), p.808
doi:10.1061/(ASCE)0733-9399(2002)128:8(808)
[9] M. Kouzeli, A. Mortensen, Acta Materialia Vol.50 (2002), p.39
doi:10.1016/S1359-6454(01)00327-5
[13] S.H. Chen, T. Wang, Acta Mechanica Vol.157 (2002), p.113
doi:10.1007/BF01182158
[17] F. Xun, G.K. Hu, Z.P. Huang, Int. J. Solids and Structures Vol.41 (2004), p.247
doi:10.1016/j.ijsolstr.2003.09.018