Available online at www.sciencedirect.com
ScienceDirect
International Journal of Solids and Structures 44 (2007) 8106–8118
INTERNATIONAL JOURNAL OF SOLIDS AND STRUCTURES
www.elsevier.com/locate/ijsolstr

Effective moduli for micropolar composite with interface effect

Huan Chen [a], Gengkai Hu [a,*], Zhuping Huang [b]

[a] *Department of Applied Mechanics, Beijing Institute of Technology, 100081 Beijing, China*
[b] *LTCS and Department of Mechanics, Peking University, Beijing 100871, PR China*

Received 22 March 2007; received in revised form 31 May 2007
Available online 9 June 2007

Abstract

Size-dependence is well observed for metal matrix composites, however the classical micromechanical model fails to describe this phenomenon. There are two different ways to consider this size-dependency: the first approach is to include the nonlocal effect by idealizing the matrix material as a high order continuum (e.g., micropolar or strain gradient); the second is to take into account the interface effect. In this work, we combine these two approaches together by introducing the interface effect into a micropolar micromechanical model. The interface constitutive relations and the generalized Young–Laplace equation for micropolar material model are firstly presented. Then they are incorporated into the micropolar micromechanical model to predict the effective bulk and shear moduli of a fiber-reinforced composite. Two intrinsic length scales appear: one is related to the microstructure of the matrix material, the other comes from the interface effect. The size-dependent effective moduli due to the nonlocal effect and interface effect can be synchronized or desynchronized for nanosize fibers, depending on the nature of the interface. For the relatively large fiber size, the size-dependence is dominated by the nonlocal effect. As expected, when the fiber size tends to infinity, classical result can be recovered.

Keywords: Micropolar; Interface effect; Composite; Size effect; Effective property

1. Introduction

Homogenization method has been recognized as a rapid developing scheme in the past decades due to a strong desire for tailoring material microstructures. Different techniques for establishing the relation between the effective property and the microstructure of a heterogeneous material are summarized in references (Nemat-Nasser and Hori, 1993; Milton, 2002; Hashin, 1983; Buryachenko, 2001; Hu and Weng, 2000). However, the classical homogenization approach fails to predict the size-dependence of the effective property, well observed in the experiment (Kouzeli and Mortensen, 2002). Since the classical methods are based on the assumptions that there is well separation of length scales and that the interfacial bonding is perfect. In order to consider the size effect, two different approaches have been proposed: one is based on the high order continuum model for constituent materials; the other argues that the interface effect comes into play. For the first

* Corresponding author. Tel./fax: +86 10 68912631.
E-mail address: hugeng@bit.edu.cn (G. Hu).

0020-7683/$ - see front matter
doi:10.1016/j.ijsolstr.2007.06.001

approach, the strain gradient (Smyshlyaev and Fleck, 1994) or micropolar (Liu and Hu, 2005; Hu et al., 2005) models have been incorporated into proper micromechanical models, the size-dependence of the overall elastic and plastic properties for composite materials can be predicted. An intrinsic length has to be introduced which is usually in micrometer scale, this length scale is believed to be related to the microstructure of the constituent materials (Hu et al., 2005). In the second approach, the constituent materials are assumed to be local in nature, however the stress discontinuity is allowed across the interface between the matrix and the reinforced phase, and this discontinuity is governed by Young–Laplace equations (Huang and Wang, 2006; Huang and Sun, 2007; Duan et al., 2005). The effective modulus predicted by the interface model varies in a complicated manner as a function of particle size, depending on the nature of the interface (e.g., Huang and Sun, 2007; Duan et al., 2005; Sharma et al., 2003). An intrinsic length scale is also introduced which is related to the property of the interface. It seems that this size effect only pronounced as the inclusions are within nanometer scales (Huang and Sun, 2007; Duan et al., 2005; Sharma et al., 2003). The scaling law for the effective modulus with the interface effect is also discussed recently (Wang et al., 2006; Duan et al., 2007).

It is of interest to examine both nonlocal effect and the interface effect, since with decreasing the size of the inclusions, these two effects become more and more pronounced. In the context of strain gradient theory, Zhang and Sharma (2005), include the interface effect to analyze the strain and stress distribution for quantum dots structure. However, the works concerned with both nonlocal and interface effects still merit further study.

In this paper, an analytical approach is proposed to include the interface effect in a micropolar micromechanical model. The manuscript is arranged as follows: the constitutive relations of the interface and the generalized Young–Laplace equations across the interface for a micropolar material are presented in Section 2, the micro–macro transition method is employed and the computation of the effective moduli of a fiber-reinforced composite is carried out in Section 3 in the framework of micropolar theory with interface effect, and the scaling law of the effective modulus is discussed in Section 4, followed by concluding remarks.

2. Constitutive relations of the interface and the generalized Young–Laplace equations in micropolar theory

The influence of interface effect on stress and strain fields was formulated by Gurtin and Murdoch (1975) in a continuum framework for elastic surface of solids, and was further generalized to the case of finite deformation by Huang and Wang (2006). The surface/interface constitutive relations together with the discontinuity conditions of the stress across the interface provide the necessary conditions for the boundary-value problem to determine the stress and strain fields with interface effect. In the micropolar material model, a surface element at a material point can transmit not only forces but also moments. So in the micropolar theory, three traditional displacements, and three extra rotations are used to describe the deformable point particles. For the micropolar theory with interface effect, additional interface constitutive relations and the jump conditions across the interface are needed.

The geometrical relations, equilibrium equations and the constitutive relations for a centro-symmetric and isotropic micropolar material in the bulk are given by Eringen (1999) and Nowacki (1986)

$$\boldsymbol{\varepsilon} = \nabla \otimes \mathbf{u} - \mathbf{e} \cdot \boldsymbol{\varphi}, \quad \mathbf{k} = \nabla \otimes \boldsymbol{\varphi}, \tag{1a}$$

$$\nabla \cdot \boldsymbol{\sigma} = 0, \quad \nabla \cdot \mathbf{m} + \mathbf{e} : \boldsymbol{\sigma} = 0, \tag{1b}$$

$$\boldsymbol{\sigma} = \lambda \mathrm{Tr}(\boldsymbol{\varepsilon})\mathbf{I} + (\mu + \kappa)\boldsymbol{\varepsilon} + (\mu - \kappa)\boldsymbol{\varepsilon}^{\mathrm{T}}, \quad \mathbf{m} = \alpha \mathrm{Tr}(\mathbf{k})\mathbf{I} + (\beta + \gamma)\mathbf{k} + (\beta - \gamma)\mathbf{k}^{\mathrm{T}}, \tag{1c}$$

where $\boldsymbol{\sigma}$ and $\mathbf{m}$ are, respectively, stress and couple stress tensors, $\boldsymbol{\varepsilon}$ and $\mathbf{k}$ are the corresponding strain and torsion tensors, $\mathbf{u}$ and $\boldsymbol{\varphi}$ are displacement and micro-rotation vectors, $\mathbf{e}$ is the permutation tensor, μ, λ are classical Lamé's constants and κ, γ, β, α are the new elastic constants introduced in micropolar theory, $\mathbf{I}$ represents the 2nd rank unit tensor in a three-dimensional space, the superscript T represents the transposition of a tensor. The corresponding boundary conditions are given by

$$\mathbf{N} \cdot \boldsymbol{\sigma} = \bar{\mathbf{t}}, \quad \mathbf{N} \cdot \mathbf{m} = \bar{\mathbf{p}} \text{ on } \partial V_\sigma, \mathbf{u} = \bar{\mathbf{u}}, \quad \boldsymbol{\varphi} = \bar{\boldsymbol{\varphi}} \text{ on } \partial V_u, \tag{2}$$

where $\bar{\mathbf{t}}$ and $\bar{\mathbf{p}}$ are prescribed force and couple on the boundary ∂V_σ, $\bar{\mathbf{u}}$ and $\bar{\boldsymbol{\varphi}}$ are prescribed displacement and micro-rotation on the boundary ∂V_u, $\mathbf{N}$ is outward unit normal vector to the boundary ∂V_σ. Note that $\boldsymbol{\sigma}$ and $\boldsymbol{\varepsilon}$ are not symmetric tensors due to the presence of the couple stress and the micro-rotation. In the following, we will derive the constitutive relations of the interface and the jump conditions across the interface for stress and couple stress.

Consider a smooth interface Γ between two solid materials Ω_1 and Ω_2 with the unit normal vector $\mathbf{n}$ directed form Ω_1 to Ω_2. Both Ω_1 and Ω_2 are centro-symmetric and isotropic micropolar materials and they satisfy the governing equations (1). In the following, subscript s represents the quantity on the interface. In the interface model, the displacement $\mathbf{u}$ and micro-rotation $\boldsymbol{\varphi}$ across the interface are assumed to be continuous, so the strain $\boldsymbol{\varepsilon}_s$ and torsion $\mathbf{k}_s$ of the interface can be defined. However the stress $\boldsymbol{\sigma}$ and couple stress $\mathbf{m}$ are discontinuous across the interface. Firstly, we construct a curvilinear coordinate system on the interface, which has covariant base vector $\mathbf{a}_\alpha$ ($\alpha = 1,2$) on the tangent plane of the interface. The unit normal vector is denoted by $\mathbf{a}_3$ or $\mathbf{n}$. The interface strain $\boldsymbol{\varepsilon}_s$ and interface torsion $\mathbf{k}_s$, which are 2nd rank tensors in a two-dimensional space, can be considered as the projection of the tensors $\boldsymbol{\varepsilon}$ and $\mathbf{k}$ in the three-dimensional space onto the tangent plane, for example, the strain tensor in a three-dimensional space can be written as

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}_s + \varepsilon^{\alpha 3}\mathbf{a}_\alpha \otimes \mathbf{a}_3 + \varepsilon^{3\beta}\mathbf{a}_3 \otimes \mathbf{a}_\beta + \varepsilon^{33}\mathbf{a}_3 \otimes \mathbf{a}_3 \quad (\alpha, \beta = 1, 2),$$

where $\boldsymbol{\varepsilon}_s = \varepsilon^{\alpha\beta}\, \mathbf{a}_\alpha \otimes \mathbf{a}_\beta$. So the interface strain $\boldsymbol{\varepsilon}_s$ and the interface torsion $\mathbf{k}_s$ can be expressed by

$$\boldsymbol{\varepsilon}_s = \mathbf{P} \cdot \boldsymbol{\varepsilon} \cdot \mathbf{P}, \quad \mathbf{k}_s = \mathbf{P} \cdot \mathbf{k} \cdot \mathbf{P}, \tag{3}$$

where $\mathbf{P} = \mathbf{I} - n \otimes \mathbf{n}$ is the projection tensor.

With the help of Eqs. (1a, 3), we obtain

$$\boldsymbol{\varepsilon}_s = \nabla_s \otimes \mathbf{u} - (\mathbf{e} \cdot \boldsymbol{\varphi})_s, \quad \mathbf{k}_s = \nabla_s \otimes \boldsymbol{\varphi}, \tag{4}$$

where $(\cdot)_s$ represents the projection operator onto the tangent plane of the interface, and ∇_s denotes the gradient operator of the interface. Supposed that the displacement $\mathbf{u}$ and the micro-rotation $\boldsymbol{\varphi}$ on the interface can be decomposed into a tangential part $\mathbf{u}_t$ and a normal part $\mathbf{u}_n$, i.e., $\mathbf{u} = \mathbf{u}_t + \mathbf{u}_n$ and $\boldsymbol{\varphi} = \boldsymbol{\varphi}_t + \boldsymbol{\varphi}_n$, where $\mathbf{u}_t = \mathbf{P} \cdot \mathbf{u}_s$, $\mathbf{u}_n = u^n\mathbf{a}_3$, $\boldsymbol{\varphi}_t = \mathbf{P} \cdot \boldsymbol{\varphi}_s$ and $\boldsymbol{\varphi}_n = \varphi^n\mathbf{a}_3$ with u^n and ϕ^n being the normal components of $\mathbf{u}$ and $\boldsymbol{\varphi}$ at the interface. Then by using the Weingarten formula, we have $\nabla_s \otimes \mathbf{u}_n = -u^n\mathbf{b}$ and $\nabla_s \otimes \boldsymbol{\varphi}_n = -\phi^n\mathbf{b}$, where $\mathbf{b}$ is the curvature tensor of the interface. Noting that, $(\mathbf{e} \cdot \boldsymbol{\varphi})_s$ can be written as

$$(\mathbf{e} \cdot \boldsymbol{\varphi})_s = \phi^n\mathbf{e}_s, \tag{5}$$

where $\mathbf{e}_s$ is the permutation tensor in the tangential plane. We obtain

$$\boldsymbol{\varepsilon}_s = \nabla_s \otimes \mathbf{u}_t - u^n\mathbf{b} - \phi^n\mathbf{e}_s, \quad \mathbf{k}_s = \nabla_s \otimes \boldsymbol{\varphi}_t - \phi^n\mathbf{b}. \tag{6}$$

There are two kinds of interface models to predict the overall properties in the existing literature. The first one is the interface energy model, and the second one is the interface stress model. It is noted that there should be a residual stress field due to the presence of the surface/interface energy (and surface/interface stresses) in materials, even though there is no external loading. In the interface energy model, the above mentioned residual surface/interface tension is taken into account. Therefore, the influence of the liquid-like residual surface tension on the effective properties of the composite materials can also be included (Huang and Sun, 2007). In the interface stress model, there is no residual stress field induced by the surface/interface tension when the material is not subjected to any external loading. So this is model is only valid in the special case where the residual surface/interface tension (or the residual surface/interface energy) can be neglected, and under the infinitesimal deformation, the interface Cauchy stress and the interface Piola-Kirchhoff stress are the same. In order to simplify the discussion, only the interface stress model will be adopted in this paper.

In the case of infinitesimal deformation, the interface constitutive relations can be expressed in terms of the interfacial free energy $\Psi_s = \Psi_s(\boldsymbol{\varepsilon}_s, \mathbf{k}_s)$

$$\boldsymbol{\sigma}_s = \frac{\partial \Psi_s}{\partial \boldsymbol{\varepsilon}_s}, \quad \mathbf{m}_s = \frac{\partial \Psi_s}{\partial \mathbf{k}_s}, \tag{7}$$

where $\boldsymbol{\sigma}_s$ and $\mathbf{m}_s$ are interface stress and interface couple stress tensors, both of which are 2nd rank tensors in a two-dimensional space.

In the following, the interface is assumed to be isotropic, and the interface constitutive relation can be written as: $\boldsymbol{\sigma}_s = \lambda_s \mathrm{Tr}(\boldsymbol{\varepsilon}_s)\ \mathbf{I}^{(2)} + 2\mu_s\boldsymbol{\varepsilon}_s$, where λ_s and μ_s are the material constants of the interface. For a centro-symmetric and isotropic material, the interface constitutive relations in the micropolar theory can be written as

$$\boldsymbol{\sigma}_s = \lambda_s \mathrm{Tr}(\boldsymbol{\varepsilon}_s)\mathbf{I}^{(2)} + (\mu_s + \kappa_s)\boldsymbol{\varepsilon}_s + (\mu_s - \kappa_s)\boldsymbol{\varepsilon}_s^{\mathrm{T}}, \tag{8a}$$

$$\mathbf{m}_s = \alpha_s \mathrm{Tr}(\mathbf{k}_s)\mathbf{I}^{(2)} + (\beta_s + \gamma_s)\mathbf{k}_s + (\beta_s - \gamma_s)\mathbf{k}_s^{\mathrm{T}}, \tag{8b}$$

where $\mathbf{I}^{(2)}$ represents the 2nd rank unit tensor in two-dimensional space.

Now, we consider a micropolar composite material with interface effect, which is subjected to a displacement boundary condition. If the body force is neglected, the total free energy of the composite material can be expressed by

$$\Pi = \int_\Gamma \Psi_s(\boldsymbol{\varepsilon}_s, \mathbf{k}_s)\mathrm{d}\Gamma + \int_{V_1+V_2} \Psi(\boldsymbol{\varepsilon}, \mathbf{k})\mathrm{d}V, \tag{9}$$

where V_1 and V_2 represent the corresponding volumes of the solid materials Ω_1 and Ω_2, respectively. The variation of the interfacial free energy Ψ_s is

$$\begin{aligned}\delta\Psi_s &= \frac{\partial\Psi_s}{\partial\boldsymbol{\varepsilon}_s} : \delta\boldsymbol{\varepsilon}_s + \frac{\partial\Psi_s}{\partial\mathbf{k}_s} : \delta\mathbf{k}_s \\ &= \nabla_s \cdot (\boldsymbol{\sigma}_s \cdot \delta\mathbf{u}_t) - (\nabla_s \cdot \boldsymbol{\sigma}_s) \cdot \delta\mathbf{u}_t - (\boldsymbol{\sigma}_s : \mathbf{b})\delta u^{\mathrm{n}} - (\boldsymbol{\sigma}_s : \mathbf{e}_s)\delta\phi^{\mathrm{n}} \\ &\quad + \nabla_s \cdot (\mathbf{m}_s \cdot \delta\boldsymbol{\varphi}_t) - (\nabla_s \cdot \mathbf{m}_s) \cdot \delta\boldsymbol{\varphi}_t - (\mathbf{m}_s : \mathbf{b})\delta\phi^{\mathrm{n}}.\end{aligned} \tag{10}$$

Consider a region enclosed by an arbitrary closed smooth curve $\partial\Gamma$ in the curved surface Γ. By using the Green-Stokes theorem, we have

$$\begin{aligned}\delta\int_\Gamma \Psi_s(\boldsymbol{\varepsilon}_s, \mathbf{k}_s)\mathrm{d}\Gamma = \int_{\partial\Gamma} \tilde{\mathbf{n}} \cdot (\boldsymbol{\sigma}_s \cdot \delta\mathbf{u}_t + \mathbf{m}_s \cdot \delta\boldsymbol{\varphi}_t)\mathrm{d}l - \int_\Gamma [(\nabla_s \cdot \boldsymbol{\sigma}_s) \cdot \delta\mathbf{u}_t + (\boldsymbol{\sigma}_s : \mathbf{b})\delta u^{\mathrm{n}} \\ + (\nabla_s \cdot \mathbf{m}_s) \cdot \delta\boldsymbol{\varphi}_t + (\mathbf{m}_s : \mathbf{b})\delta\phi^{\mathrm{n}} + (\boldsymbol{\sigma}_s : \mathbf{e}_s)\delta\phi^{\mathrm{n}}]\mathrm{d}\Gamma,\end{aligned} \tag{11}$$

where $\mathrm{d}l$ is the element of the arc length on $\partial\Gamma$, $\tilde{\mathbf{n}}$ is outward unit normal vector to the curve $\partial\Gamma$. The variation of the second term on the right hand side of Eq. (9) can be written as

$$\begin{aligned}\delta\int_V \Psi(\boldsymbol{\varepsilon}, \mathbf{k})\mathrm{d}V = -\int_\Gamma (\mathbf{n} \cdot [\boldsymbol{\sigma}] \cdot (\delta\mathbf{u}_t + \delta\mathbf{u}_n) + \mathbf{n} \cdot [\mathbf{m}] \cdot (\delta\boldsymbol{\varphi}_t + \delta\boldsymbol{\varphi}_n))\mathrm{d}\Gamma \\ - \int_{V_1+V_2} (\nabla \cdot \boldsymbol{\sigma}) \cdot \delta\mathbf{u}\mathrm{d}V - \int_{V_1+V_2} (\nabla \cdot \mathbf{m} + \boldsymbol{\sigma} : \mathbf{e}) \cdot \delta\boldsymbol{\varphi}\mathrm{d}V,\end{aligned} \tag{12}$$

where $[\cdot]$ represents the jump of stress or couple stress across the interface, e.g., $[\boldsymbol{\sigma}] = \boldsymbol{\sigma}^2 - \boldsymbol{\sigma}^1$. $\mathbf{n}$ is unit normal vector to the interface Γ, directed from Ω_1 to Ω_2. The minimum potential energy requires $\delta\Pi = 0$. From the arbitrariness of $\delta\mathbf{u}_t$, δu^{n}, $\delta\boldsymbol{\varphi}_t$ and $\delta\phi^{\mathrm{n}}$, it can be seen that the vanishing of the variation of Eq. (9) leads to the following generalized Young–Laplace equation:

$$\mathbf{n} \cdot [\boldsymbol{\sigma}] \cdot \mathbf{P} = -\nabla_s \cdot \boldsymbol{\sigma}_s, \quad \mathbf{n} \cdot [\boldsymbol{\sigma}] \cdot \mathbf{n} = -\boldsymbol{\sigma}_s : \mathbf{b}, \tag{13a}$$

$$\mathbf{n} \cdot [\mathbf{m}] \cdot \mathbf{P} = -\nabla_s \cdot \mathbf{m}_s, \quad \mathbf{n} \cdot [\mathbf{m}] \cdot \mathbf{n} = -(\mathbf{m}_s : \mathbf{b} + \boldsymbol{\sigma}_s : \mathbf{e}_s). \tag{13b}$$

Eq. (13a) is the jump condition in the classical continuum mechanics with interface effect, and the second Eq. (13b) corresponds to the one in the micropolar theory. From the above discussion, we can see that the elastic fields are governed by Eq. (1), the interface constitutive relations (8) and the generalized Young–Laplace equations (13), if a proper boundary condition is prescribed. In the following, we will apply the above discussions to predict the effective moduli of a fiber-reinforced composite.

3. Effective elastic properties of micropolar composites with interface effect

3.1. Theoretical formulation

As in the references (Liu and Hu, 2005; Hu et al., 2005; Xun et al., 2004), here we are interested in the classical effective property of a micropolar composite, which are related to the average symmetric stress and strain over representative volume element (RVE) by $<\boldsymbol{\sigma}^{\rm sym}>=\bar{\mathbf{C}}^{\rm sym}:<\boldsymbol{\varepsilon}^{\rm sym}>$ or $<\boldsymbol{\varepsilon}^{\rm sym}>=\bar{\mathbf{M}}^{\rm sym}:<\boldsymbol{\sigma}^{\rm sym}>$. The superscript sym means symmetric part of the corresponding quantity. To this end, we follow the method proposed by Liu and Hu (2005). Consider a RVE consisting of a two-phase material occupying a volume V with the boundary ∂V. The volumes of the reinforced phase Ω_1 and the matrix Ω_2 are V_1 and V_2, respectively, and $\mathbf{N}$ is outward unit normal vector to ∂V. The micropolar composite is assumed to be centro-symmetric with the effective modulus (compliances) $\bar{\mathbf{C}}_3(\overline{M}_3)$, and the moduli (compliances) of the reinforced phase and the matrix are denoted by $\mathbf{C}_1(M_1)$ and $\mathbf{C}_2(M_2)$, respectively. The modulus tensor can be decomposed into symmetric and anti-symmetric parts as $\mathbf{C}_i=\mathbf{C}_i^{\rm sym}+\mathbf{C}_i^{\rm asym}$ $(i=1,2,3)$. The following boundary condition for the RVE will be adopted

$$\mathbf{N}\cdot\boldsymbol{\sigma}^{\rm sym}=\mathbf{N}\cdot\boldsymbol{\Sigma}^{\rm sym},\quad \mathbf{N}\cdot\mathbf{m}=0. \tag{14}$$

This special boundary condition will allow one to derive the classical (the symmetric part) moduli of the composite (Liu and Hu, 2005; Hu et al., 2005).

According to Benveniste and Miloh (2001), the symmetric average stress and average strain are related to the remotely applied stress or strain on the boundary of the RVE as follows:

$$<\boldsymbol{\sigma}^{\rm sym}>=\frac{1}{2V}\int_S\left\{\left(\mathbf{N}\cdot\sum^{\rm sym}\right)\otimes\mathbf{x}+\mathbf{x}\otimes\left(\mathbf{N}\cdot\sum^{\rm sym}\right)\right\}{\rm d}S, \tag{15}$$

$$<\boldsymbol{\varepsilon}^{\rm sym}>=\frac{1}{2V}\int_S(\mathbf{u}\otimes\mathbf{N}+\mathbf{N}\otimes\mathbf{u}){\rm d}S. \tag{16}$$

In the interface stress model, the displacement is continuous, but the stress has a jump across the interface between the matrix and the reinforced phase. So the average symmetric strain and stress over the RVE can be expressed by

$$<\boldsymbol{\sigma}^{\rm sym}>=(1-f)<\boldsymbol{\sigma}^{\rm sym}>_2+f<\boldsymbol{\sigma}^{\rm sym}>_1+\frac{f}{2V_1}\int_\Gamma\{(\mathbf{n}\cdot[\boldsymbol{\sigma}^{\rm sym}])\otimes\mathbf{x}+\mathbf{x}\otimes(\mathbf{n}\cdot[\boldsymbol{\sigma}^{\rm sym}])\}{\rm d}\Gamma, \tag{17}$$

$$<\boldsymbol{\varepsilon}^{\rm sym}>=(1-f)<\boldsymbol{\varepsilon}^{\rm sym}>_2+f<\boldsymbol{\varepsilon}^{\rm sym}>_1, \tag{18}$$

where $<\cdot>_i$ denotes the volume average of the said quantity over the region i $(i=1,2)$. f is the volume fraction of the reinforced phase.

For the applied macroscopic loading condition Eq. (14), it can be shown that

$$\begin{aligned}<\boldsymbol{\sigma}^{\rm sym}>&=(1-f)<\boldsymbol{\sigma}^{\rm sym}>_2+f<\boldsymbol{\sigma}^{\rm sym}>_1+\frac{f}{2V_1}\int_\Gamma\{(\mathbf{n}\cdot[\boldsymbol{\sigma}^{\rm sym}])\otimes\mathbf{x}+\mathbf{x}\otimes(\mathbf{n}\cdot[\boldsymbol{\sigma}^{\rm sym}])\}{\rm d}\Gamma\\&=\frac{1}{2V}\left\{\int_S(\mathbf{N}\cdot\boldsymbol{\Sigma}^{\rm sym})\otimes\mathbf{x}{\rm d}S+\int_S\mathbf{x}\otimes(\mathbf{N}\cdot\boldsymbol{\Sigma}^{\rm sym}){\rm d}S\right\}=\boldsymbol{\Sigma}^{\rm sym}.\end{aligned} \tag{19}$$

The symmetric stress concentration tensors $\mathbf{P}_i^{\rm sym}$ $(i=1,2)$ for the different phases and symmetrical stress concentration tensors $\mathbf{P}_{\rm s}^{\rm sym}$ for the interface can be defined by

$$<\boldsymbol{\sigma}^{\rm sym}>_i=\mathbf{P}_i^{\rm sym}:\boldsymbol{\Sigma}^{\rm sym}, \tag{20}$$

$$\frac{1}{2V_1}\int_\Gamma\{(\mathbf{n}\cdot[\boldsymbol{\sigma}^{\rm sym}])\otimes\mathbf{x}+\mathbf{x}\otimes(\mathbf{n}\cdot[\boldsymbol{\sigma}^{\rm sym}])\}{\rm d}\Gamma=\mathbf{P}_{\rm s}^{\rm sym}:\boldsymbol{\Sigma}^{\rm sym}. \tag{21}$$

With the help of Eqs. (17)–(21), the symmetric part of the effective compliance tensor of the micropolar composite can be derived as

$$\bar{\mathbf{M}}_3^{\rm sym}=(1-f)\mathbf{M}_2^{\rm sym}:\mathbf{P}_2^{\rm sym}+f\mathbf{M}_1^{\rm sym}:\mathbf{P}_1^{\rm sym}, \tag{22a}$$

or

$$\bar{\mathbf{M}}_3^{\rm sym}=\mathbf{M}_2^{\rm sym}+f(\mathbf{M}_1^{\rm sym}-\mathbf{M}_2^{\rm sym}):\mathbf{P}_1^{\rm sym}-f\mathbf{M}_2^{\rm sym}:\mathbf{P}_{\rm s}^{\rm sym}. \tag{22b}$$

It can be seen that once $\mathbf{P}_i^{\rm sym}$ and $\mathbf{P}_{\rm s}^{\rm sym}$ are obtained, Eq. (22) can be used to evaluate the classical effective moduli of the micropolar composite. Different methods can be used to estimate the concentration tensors

$\mathbf{P}_i^{\text{sym}}$ and $\mathbf{P}_{\text{s}}^{\text{sym}}$, e.g., Mori-Tanaka method (MTM) or the generalized self-consistent method (GSCM), just as in the classical micromechanics.

In the following, we will focus on the discussion of a two-dimensional fiber composite, and will derive the size-dependent effective in plane bulk and shear moduli with interface effect.

3.2. Applications to fiber-reinforced composites

For a two-dimensional composite with cylindrical fibers, the in-plane effective bulk and shear moduli have been derived by Xun et al. (2004) in the framework of micropolar theory without interface effect. In this paper, we will follow the same approach and include the interface effect. For the two-dimensional problem, the governing equations are

$$\varepsilon_{\beta\alpha} = u_{\alpha,\beta} + e_{\alpha\beta3}\varphi_3, \quad \kappa_{\alpha3} = \varphi_{3,\alpha}, \tag{23a}$$

$$\sigma_{\beta\alpha,\beta} = 0, \quad m_{\rho3,\rho} + e_{3\alpha\beta}\sigma_{\alpha\beta} = 0, \tag{23b}$$

$$\sigma_{\beta\alpha} = \lambda\varepsilon_{\rho\rho}\delta_{\beta\alpha} + (\mu+\kappa)\varepsilon_{\beta\alpha} + (\mu-\kappa)\varepsilon_{\alpha\beta}, \quad m_{\alpha3} = (\beta+\gamma)\kappa_{\alpha3}. \tag{23c}$$

where subscripts α and β range from 1 to 2.

The corresponding boundary conditions are

$$\sigma_{\beta\alpha}n_\beta = \tilde{\sigma}_\alpha, \quad m_{\alpha3}n_\alpha = \tilde{m}_3 \text{ on } \Gamma^\sigma, \tag{24a}$$

$$u_\alpha = \tilde{u}_\alpha, \quad \varphi_\alpha = \tilde{\varphi}_\alpha \text{ on } \Gamma^u. \tag{24b}$$

In the cylindrical coordinate, the interface constitutive relations can be written as

$$\sigma^{\text{s}}_{\theta\theta} = (\lambda_{\text{s}} + 2\mu_{\text{s}})\varepsilon^{\text{s}}_{\theta\theta}, \quad m^{\text{s}}_{\theta z} = (\beta_{\text{s}} + \gamma_{\text{s}})\kappa^{\text{s}}_{\theta z}. \tag{25}$$

In order to determine the effective property of the composite, let us consider the following problem: a cylindrical fiber with a matrix coating is embedded in an infinite host material, and it is subjected to a uniform remote traction. The fiber radius is denoted by R_1 and the radius of the matrix coating by R_2. Following the approach employed in reference (Xun et al., 2004), the stress and the couple stress in the cylindrical coordinate can be expressed in terms of the potential functions F_i, G_i in the region i ($i = 1, 2, 3$, respectively, representing fiber, matrix and another infinite host material)

$$\sigma^i_{rr} = \frac{1}{r}\frac{\partial F_i}{\partial r} + \frac{1}{r^2}\frac{\partial^2 F_i}{\partial\theta^2} - \frac{1}{r}\frac{\partial^2 G_i}{\partial r\partial\theta} + \frac{1}{r^2}\frac{\partial G_i}{\partial\theta}, \tag{26a}$$

$$\sigma^i_{\theta\theta} = \frac{1}{r^2}\frac{\partial^2 F_i}{\partial r^2} + \frac{1}{r}\frac{\partial^2 G_i}{\partial r\partial\theta} - \frac{1}{r^2}\frac{\partial G_i}{\partial\theta}, \tag{26b}$$

$$\sigma^i_{r\theta} = \frac{1}{r}\frac{\partial^2 F_i}{\partial r\partial\theta} + \frac{1}{r^2}\frac{\partial F_i}{\partial\theta} - \frac{1}{r}\frac{\partial G_i}{\partial r} - \frac{1}{r^2}\frac{\partial^2 G_i}{\partial\theta^2}, \tag{26c}$$

$$m^i_{rz} = \frac{\partial G_i}{\partial r}, \tag{26d}$$

$$m^i_{\theta z} = \frac{1}{r}\frac{\partial G_i}{\partial\theta}. \tag{26e}$$

These potentials can be derived from the following governing equations:

$$\frac{\partial}{\partial r}(G_i - {l^i_{\text{m}}}^2\nabla^2 G_i) = -2(1-v_i)b_i\frac{1}{r}\frac{\partial}{\partial\theta}(\nabla^2 F_i), \tag{27a}$$

$$\frac{1}{r}\frac{\partial}{\partial\theta}(G_i - {l^i_{\text{m}}}^2\nabla^2 G_i) = 2(1-v_i)b_i\frac{\partial}{\partial r}(\nabla^2 F_i), \tag{27b}$$

where $l^i_{\text{m}} = \sqrt{\frac{(\beta_i+\gamma_i)(\kappa_i+\mu_i)}{4\mu_i\kappa_i}} = b_i\sqrt{\frac{\kappa_i+\mu_i}{\kappa_i}}$, $v_i = \frac{\lambda_i}{2(\lambda_i+\mu_i)}$, and the constants l^i_{m} or b_i are the intrinsic length scales for the micropolar materials. Eq. (27) can also be written as

$$\nabla^4 F_i = 0, \tag{28a}$$

$$\nabla^2(G_i - l_{\mathrm{m}}^{i\,2}\nabla^2 G_i) = 0, \tag{28b}$$

where ∇^2 is the Laplacian operator. The general solutions of Eq. (28) are given by

$$F_i = A_1^i R_1^2 \log r + A_2^i r^2 + (A_3^i R_1^2 + A_4^i r^2 + A_5^i R_1^4 r^{-2} + A_6^i R_1^{-2} r^4)\cos 2\theta, \tag{29a}$$

$$G_i = [A_7^i R_1^4 r^{-2} + A_8^i r^2 + A_9^i R_1^2 K_2(r/l_{\mathrm{m}}^i) + A_{10}^i R_1^2 I_2(r/l_{\mathrm{m}}^i)]\sin 2\theta, \tag{29b}$$

where $I_{\mathrm{M}}(r/l_{\mathrm{m}}^i)$ and $K_{\mathrm{M}}(r/l_{\mathrm{m}}^i)$ are the first type and the second type modified Bessel functions. The constants A_j^i can be determined by the interface conditions and the remote boundary condition. In the following, two micromechanical models will be used to derive the effective moduli of the composite material, namely the MTM and GSCM. For the MTM, the infinite host material is assumed to be the same as the matrix material. For the GSCM, we set $R_1^2/R_2^2 = f$ and the infinite host material is taken to be the yet-unknown composite material. Both for the MTM and GSCM, the stress and couple stress jump conditions across the interface between the fiber and the matrix at $r = R_1$ are given by Eqs. (13a, 13b). For the GSCM, a perfectly bonded interface is assumed between the matrix and the unknown composite ($r = R_2$). The continuity conditions of the displacement and rotation, and the jump conditions for the stress and couple stress at $r = R_1$ can be expressed by

$$u_r^1(R_1) = u_r^2(R_1), \quad u_\theta^1(R_1) = u_\theta^2(R_1), \quad \varphi_z^1(R_1) = \varphi_z^2(R_1), \tag{30a}$$

$$\sigma_{rr}^2 - \sigma_{rr}^1 = \left(\frac{1}{R_1}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{R_1}\right)(\lambda_{\mathrm{s}} + 2\mu_{\mathrm{s}})/R_1, \tag{30b}$$

$$\sigma_{r\theta}^2 - \sigma_{r\theta}^1 = -\frac{\partial}{\partial \theta}\left[\left(\frac{1}{R_1}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{R_1}\right)(\lambda_{\mathrm{s}} + 2\mu_{\mathrm{s}})/R_1\right], \tag{30c}$$

$$m_{rz}^2 - m_{rz}^1 = -\frac{\partial}{\partial \theta}\left(\frac{1}{R_1}\frac{\partial \phi_z}{\partial \theta}\right)(\beta_{\mathrm{s}} + \gamma_{\mathrm{s}})/R_1. \tag{30d}$$

In the GSCM, the interface conditions at $r = R_2$ are given by

$$u_r^2(R_2) = u_r^3(R_2), \quad u_\theta^2(R_2) = u_\theta^3(R_2), \quad \varphi_z^2(R_2) = \varphi_z^3(R_2), \tag{31a}$$

$$\sigma_{rr}^2(R_2) = \sigma_{rr}^3(R_2), \quad \sigma_{r\theta}^2(R_2) = \sigma_{r\theta}^3(R_2), \quad m_{rz}^2(R_2) = m_{rz}^3(R_2). \tag{31b}$$

In this paper, the fiber material is assumed to be classical Cauchy material and the effective composite is also considered as a classical Cauchy material (Hu et al., 2005). In this case, the fiber imposes zero micro-rotation at the interface. Hence only the matrix material is of micropolar type with κ, l_{m} as additional elastic constants.

3.2.1. Bulk modulus of the fiber composite

In order to predict the effective bulk modulus of the composite, it is convenient to apply a hydrostatic stress on the remote boundary, i.e., $\Sigma_{xx} = \Sigma_{yy} = \Sigma$. Both for the MTM and GSCM, the unknown constants are determined from the corresponding interface conditions and the remote boundary condition. After some tedious mathematical calculations, it is found that the bulk moduli determined by the MTM and GSCM are the same, and can be expressed as

$$\frac{\bar{k}_3}{k_2} = \frac{2k_1(1 + f\mu_2/k_2) + \mu_2[2(1-f) + (l_{\mathrm{s}}/R_1)(1 + f\mu_2/k_2)]}{2(1-f)k_1 + 2fk_2 + [2 + (l_{\mathrm{s}}/R_1)(1-f)]\mu_2}, \tag{32}$$

where $l_{\mathrm{s}} = (\lambda_{\mathrm{s}} + 2\mu_{\mathrm{s}})/\mu_2$, denoting the intrinsic length related to the interface. As expected, the micropolar theory gives the same result as the classical model, since the micropolar theory only involves a rigid micro-rotation of a material point. The result given by Eq. (32) is the same as that in the reference (Chen et al., 2007; Karihaloo et al., 2006) for the classical material model with interface effect.

3.2.2. In-plane effective shear modulus of the fiber composite

In order to obtain the effective in-plane shear modulus of the composite, a pure shear loading is applied on the remote boundary, i.e., $\Sigma_{xx} = -\Sigma_{yy} = \Sigma$. Due to the complicated nature of the micropolar theory, the

GSCM can not deliver a closed-form expression for the effective shear modulus. However, a closed-form expression of the effective shear modulus can be derived by the MTM, and it is given by

$$\frac{\bar{\mu}_3}{\mu_2} = \frac{\zeta_0 + \zeta_1 l_m/R_1 + \zeta_2 l_m l_s/R_1^2 + \zeta_3 l_s/R_1}{\zeta_0' + \zeta_1 l_m/R_1 + \zeta_2 l_m l_s/R_1^2 + \zeta_3' l_s/R_1}, \tag{33}$$

where

$$\begin{aligned}
&\zeta_0 = H_0 K_2(R_1/l_m), \quad \zeta_1 = H_1 K_1(R_1/l_m), \quad \zeta_2 = H_2 K_1(R_1/l_m),\\
&\zeta_3 = H_3 K_2(R_1/l_m), \quad \zeta_0' = H_4 K_2(R_1/l_m), \quad \zeta_3' = H_5 K_2(R_1/l_m).\\
&H_0 = -2[2\mu_1\mu_2 + k_1(\mu_1+\mu_2)][2\mu_1\mu_2 + k_2[\mu_1(1+f) + \mu_2(1-f)]],\\
&H_1 = \frac{1}{\kappa+\mu_2}[4\kappa(1-f)(\mu_1-\mu_2)(k_2+\mu_2)(2\mu_1\mu_2 + k_1(\mu_1+\mu_2))],\\
&H_2 = \frac{1}{\kappa+\mu_2}[2\kappa\mu_2(1-f)(k_2+\mu_2)[k_1(2\mu_1-\mu_2) + \mu_1(3\mu_1-2\mu_2)]],\\
&H_3 = -\mu_2[\mu_1[2\mu_2(3\mu_1+\mu_2) + k_2[3(1+f)\mu_1 + 2(2-f)\mu_2]] + k_1[\mu_2(4\mu_1+\mu_2)\\
&\qquad + k_2[2(1+f)\mu_1 + (2-f)\mu_2]]],\\
&H_4 = H_0\frac{2\mu_2(\mu_1 - f\mu_1 + f\mu_2) + k_2(\mu_1 - f\mu_1 + \mu_2 + f\mu_2)}{2\mu_1\mu_2 + k_2(\mu_1 + f\mu_1 + \mu_2 - f\mu_2)},\\
&H_5 = -\mu_2[(1-f)k_2\mu_1(2k_2+3\mu_1) + \mu_2[(2+f)k_1k_2 + 4(1-f)k_1\mu_1 + 2\mu_1[(2+f)k_2\\
&\qquad + 3(1-f)\mu_1]] + (1+2f)(k_1+2\mu_1)\mu_2^2].
\end{aligned}$$

It is easy to check that when λ_s and μ_s tend to zero, the result (33) reduces to the effective shear modulus of micropolar theory without interface effect (Xun et al., 2004)

$$\frac{\bar{\mu}_3}{\mu_2} = \frac{\zeta_0 + \zeta_1 l_m/R_1}{\zeta_0' + \zeta_1 l_m/R_1}. \tag{34}$$

When neglecting the nonlocal effect, the classical result with the interface effect can be recovered

$$\frac{\bar{\mu}_3}{\mu_2} = \frac{\zeta_0 + \zeta_3 l_s/R_1}{\zeta_0' + \zeta_3' l_s/R_1} = \frac{H_0 + H_3 l_s/R_1}{H_4 + H_5 l_s/R_1}. \tag{35}$$

When R_1 tends to infinity, the effective shear modulus reduces to the classical one without the surface effect, namely $\bar{\mu}_3/\mu_2 = H_0/H_4$.

3.3. Numerical examples

In the following, some numerical calculations are performed in order to illustrate the previous theoretical prediction. An aluminum metal containing cylindrical voids ($\mu_1 = k_1 = 0$, $f = 0.2$) is chosen as the sample material, and the matrix material constants are $\mu_2 = 34.7$ GPa, $\nu_2 = 0.3$. High-order material constant $\kappa = 34.7$ GPa is assumed for the matrix material. The free-surface properties are taken from the paper of Sharma et al. (2003). Two sets of the surface moduli are examined, namely, I: $\lambda_s = 6.842$ N/m, $\mu_s = -0.3755$ N/m for the surface [1 1 1]; II: $\lambda_s = 3.48912$ N/m, $\mu_s = -6.2178$ N/m for the surface [1 0 0]. The intrinsic lengths of the interface are $l_s = 0.18$ nm for type I, and $l_s = -0.26$ nm for type II.

The variation of the effective bulk modulus as the function of void radius is shown in Fig. 1, the classical modulus without surface effect is independent of void radius, however the effective modulus involving surface effect becomes sensitive to void radius at nano-scale, it will decrease or increase with decreasing the void radius, depending on the nature of the surface. It is noted that both for the micropolar theory and classical continuum mechanics give the same bulk modulus.

The effective shear modulus involves two intrinsic length scales, namely l_m and l_s. In order to facilitate the following analysis, let $l_m = \delta|l_s|$. The variations of the effective shear modulus as the function of void radius are shown in Figs. 2 and 3 for the two types of the surface. For the nonlocal effect (micropolar), the predicted

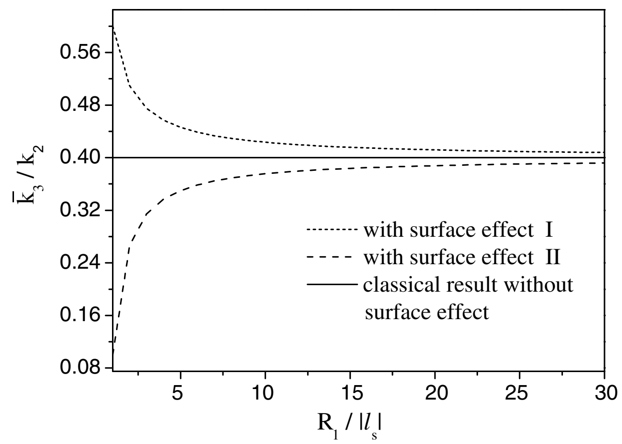

Fig. 1. Variation of effective modulus as function of void radius for two types of surface properties.

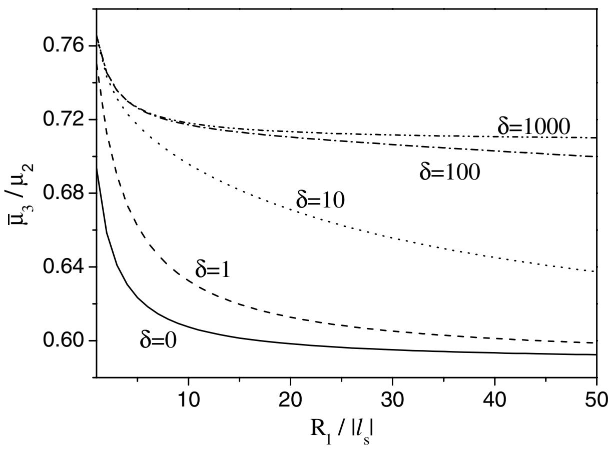

Fig. 2. Effective shear modulus as function of void radius for the surface of type I for different values δ.

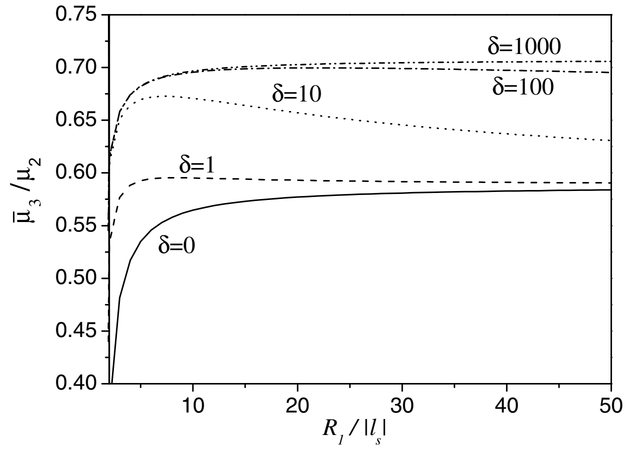

Fig. 3. Effective shear modulus as function of void radius for the surface of type II for different values δ.

effective modulus always increases with decreasing void size. For the surface of type I, nonlocal effect and surface effect are synchronized, and the effective shear modulus increases with the decreasing of the void size. It is found also that with the increase of δ, the size influential zone becomes large, and the size-effect is dominated by the nonlocal effect. For the surface of type II, the size-dependency due to the nonlocal and surface effect are desynchronized, and the present theory (micropolar with interface effect) predicts a decreasing effective shear modulus when the void size is smaller than a critical value. For the large void size, again the nonlocal effect dominates, and this is clearly illustrated in Fig. 4.

Fig. 5 illustrates the effective in-plane shear modulus predicted by the MTM and GSCM, respectively, for two void volume fractions for the surface of type I. The material constants are the same as those in the previous example, but δ is taken to be 1. It is seen that both methods predict the same trend and the classical results can be recovered when the size of the void becomes large. The same results can also be found for the surface of the type II.

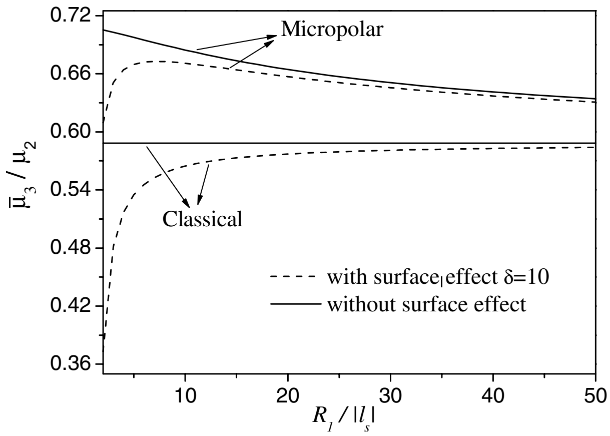

Fig. 4. Variation of the effective shear modulus as function of void radius predicted by different models with or without surface effect ($\delta = 10$), for the surface of the type II.

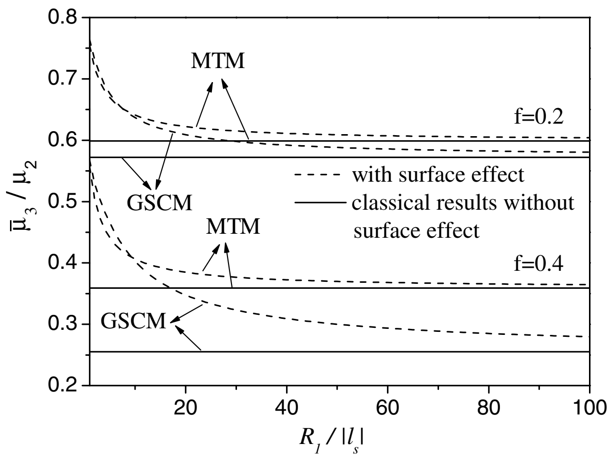

Fig. 5. The shear modulus of composite with the surface of type I predicted by MTM and GSCM ($\delta = 1$) for two different void volume fractions.

4. Scaling law

Let us consider the material property with length scale L, described by $F(L)$. In the classical material model with interface effect, Wang et al. (2006) and Duan et al. (2007) found that the ratio of $F(L)$ at small scale L to $F(\infty)$ can be written as

$$\frac{F(L)}{F(\infty)} = F\left(X_j, \frac{l_{\text{in}}}{L}\right), \tag{36}$$

where l_{in} is the intrinsic length scale of the material (it is related to the interface effect in Wang et al. (2006) and Duan et al. (2007)), L is the characteristic size of the material, and X_j represent some parameters. For a composite material, L denotes the radius of inhomogeneity R, Eq. (36) can be further expanded as in the case of $l_{\text{in}}/R < 1$

$$\frac{F(R)}{F(\infty)} = 1 + \chi\frac{l_{\text{in}}}{R} + \mathrm{O}\left(\frac{l_{\text{in}}}{R}\right)^2. \tag{37}$$

Now we will examine the corresponding scaling law for the effective shear modulus with both interface and nonlocal effects. There are two intrinsic length scales, and usually l_s is in the order of nanometer and in polycrystalline metals l_m has the order of grain size, i.e., micrometer. In the following we suppose that $l_s/R < 1$, and l_m/R can be either greater or less than unity.

(1) Case $l_s/R < 1$, $l_m/R < 1$

In this case, we have

$$\frac{F(L)}{F(\infty)} = 1 + \chi_1\frac{l_m}{L} + \chi_2\frac{l_s}{L} + \mathrm{O}\left(\frac{l_m}{L}, \frac{l_s}{L}\right)^2. \tag{38}$$

From Eq. (33) and by dropping the high order terms, we can obtain the following scaling law:

$$\frac{\mu(R)}{\mu(\infty)} = 1 + \frac{H_1(H_4 - H_0)}{H_4H_0}\frac{l_m}{R} + \frac{(H_3H_4 - H_0H_5)}{H_4H_0}\frac{l_s}{R}, \tag{39}$$

When $l_m = 0$, the scaling law for effective shear modulus predicted by classical material model with interface effect is retrieved, and when $l_s = 0$, Eq. (39) gives the scaling law for effective shear modulus predicted by micropolar material model without interface effect.

(2) Case $l_s/R < 1$, $R/l_m < 1$

In this case, we expand Eq. (33) at the point $R/l_m = 1$. From Eq. (33), and by dropping the high order terms, we have the following scaling law:

$$\frac{\mu(R)}{\mu(\infty)} = \chi_0 + \chi_1\left(\frac{R}{l_m} - 1\right) + \chi_2\frac{l_s}{R}, \tag{40}$$

where

$$\chi_0 = \frac{H_4[H_1K_1(1) + H_0K_2(1)]}{H_0[H_1K_1(1) + H_4K_2(1)]},$$

$$\chi_1 = \frac{H_1H_4(H_4 - H_0)[K_1^2(1) - K_0(1)K_2(1) - 2K_1(1)K_2(1) - K_2^2(1) + K_1(1)K_3(1)]}{2H_0[H_1K_1(1) + H_4K_2(1)]^2},$$

$$\chi_2 = \frac{H_4K_2(1)[[H_2H_4 - H_0H_2 + H_1[H_3 - H_5]]K_1(1) + [H_3H_4 - H_0H_5]K_2(1)]}{H_0[H_1K_1(1) + H_4K_2(1)]^2}.$$

It is found that when the high-order elastic constant κ vanishes, the second term of the right hand side of Eq. (40) is zero, χ_0 reduces to the unity and the coefficient of l_s/R in Eq. (40) is equal to that in Eq. (39). This means the classical result with interface effect is recovered, as expected. As an approximation of the general expression (33), the accuracy of Eq. (40) is displayed in Fig. 6. It is found that there is some discrepancy for $R/l_s \to 1$, when R/l_s increases, this approximation agrees well with the exact solution.

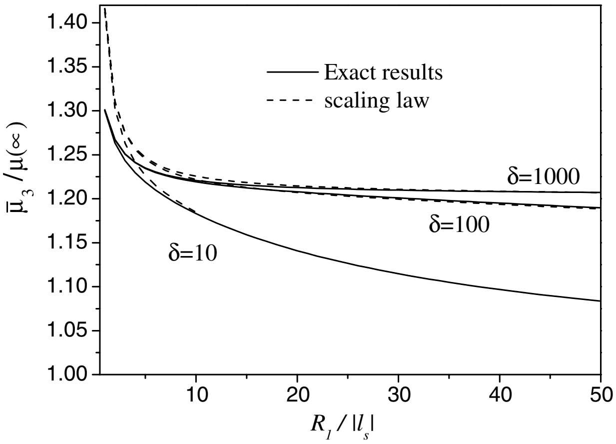

Fig. 6. Comparison with the scaling law with the exact solution for different δ in the case of $l_s/R < 1$, $R/l_m < 1$.

5. Concluding remarks

We have formulated a theoretical framework to examine the size effect due to both nonlocal effect and interface effect for a composite material. The nonlocal effect is considered by idealizing the matrix material as a micropolar material model. The interface constitutive relations and the generalized Young–Laplace equations for a micropolar material with interface effect are presented. A micropolar micromechanics with interface effect is employed to predict the effective moduli of a fiber-reinforced composite material. The effective bulk modulus is found to be the same as that predicted by the classical micromechanics with interface effect. There are two intrinsic length scales for the effective shear modulus, one comes from nonlocal effect, and the other comes from the interface effect. It is found that at nano-scale both nonlocal and surface effects dominate the size-dependent effective property of the composite. With the increase of the fiber size, the nonlocal effect becomes a dominant mechanism. When the fiber size tends to infinity, classical result will be recovered.

Acknowledgements

This work was supported by the National Natural Science Foundation of China under the Grants 10325210 and 10332020. ZP Huang thanks the support of Shanghai Leading Academic Discipline.

References

Benveniste, Y., Miloh, T., 2001. Imperfect soft and stiff interfaces in two-dimensional elasticity. Mechanic of Materials 33, 309–323.

Buryachenko, V.A., 2001. Multiparticle effective field and related methods in micromechanics of compostes materials. Applied Mechanics Review 54, 1–47.

Chen, T., Dovrak, G.J., Yu, C.C., 2007. Size-dependent elastic properties of unidirectional nano-composites with interface stresses. Acta Mechanica 188, 39–54.

Duan, H.L., Wang, J., Huang, Z.P., Karihaloo, B.L., 2005. Size-dependent effective elastic constants of solids containing nano-inhomogeneities with interface stress. Journal of the Mechanics and Physics of Solids 53, 1574–1596.

Duan, H.L., Yi, X., Huang, Z.P., Wang, J., 2007. A unified scheme for prediction of effective moduli of multiphase composites with interface effects. Part II: application and scaling laws. Mechanics of Materials 39, 94–103.

Eringen, A.C., 1999. Microcontinuum Field Theory. Springer.

Gurtin, M.E., Murdoch, A.I., 1975. A continuum theory of elastic material surfaces. Archive for Rational Mechanics and Analysis 57, 291–323.

Hashin, Z., 1983. Analysis of composites: a survey. Journal of Applied Mechanics 50, 481–505.

Hu, G.K., Weng, G.J., 2000. Connection between the double inclusion model and the Ponte Castaneda-Willis, Mori-Tanaka, and Kuster-Toksoz model. Mechanics of Materials 32, 495–503.

Huang, Z.P., Sun, L., 2007. Size-dependent effective properties of a heterogeneous material with interface energy effect: from finite deformation theory to infinitesimal strain analysis. Acta Mechanica 190, 151–163.

Huang, Z.P., Wang, J., 2006. A theory of hyperelasticity of multi-phase media with surface/interface energy effect. Acta Mechanica 182, 195–210.

Hu, G.K., Liu, X.N., Lu, T.J., 2005. A Variational method for nonlinear micropolar composite. Mechanics of Materials 37, 407–425.

Karihaloo, B.L., Duan, H.L., Wang, J., Huang, Z.P., 2006. Nanoporous materials can be made stiffer than non-porous counterparts by surface modification. Acta Materialia 54, 2983–2990.

Kouzeli, M., Mortensen, A., 2002. Size dependent strengthening in particle reinforced aluminium. Acta Materialia 50, 39–51.

Liu, X.N., Hu, G.K., 2005. A continuum micromechanical theory of overall plasticity for particulate composites including particle size effect. International Journal of Plasticity 21, 777–799.

Milton, G.W., 2002. The theory of Composite. Cambridge University Press, Cambridge, England.

Nemat-Nasser, S., Hori, M., 1993. Micromechanics: Overall Properties of Heterogeneous Materials. Elsevier, North-Holland.

Nowacki, W., 1986. Theory of Asymmetric Elasticity. Pergamon Press.

Sharma, P., Ganti, S., Bhate, N., 2003. Effect of surfaces on the size-dependent elastic state of nano-inhomogeneities. Applied Physical Letter 82, 535–537.

Smyshlyaev, V.P., Fleck, N.A., 1994. Bounds and estimates for linear composites with strain gradient effects. Journal of the Mechanics and Physics of Solids 42, 1851–1882.

Wang, J., Duan, H.L., Huang, Z.P., Karihaloo, B.L., 2006. A scaling law for properties of nano-structured materials. Proceedings of the Royal Society A 462, 1355–1363.

Xun, F., Hu, G.K., Huang, Z.P., 2004. Effective in-plane moduli of composites with a micropolar matrix and coated fibers. International Journal of Solids and Structures 41, 247–265.

Zhang, X., Sharma, P., 2005. Size dependency of strain in arbitrary shaped anisotropic embedded quantum dots due to nanlocal dispersive effects. Physical Review B 72, 195345.