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Grating effect in negative permeability meta-material

Xiaobing Cai [a], Gengkai Hu [a,b,*]

[a] School of Science, Beijing Institute of Technology, Beijing 100081, China
[b] National Key Laboratory for the Prevention and Control of Explosion Disaster, Beijing Institute of Technology, Beijing 100081, China

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Abstract

Abnormal transmission is found in meta-material with negative permeability, this transmission is attributed to grating effect. A grating model with an effective negative permeability material is proposed, it is found that indeed the grating model can give a good prediction on the unusual transmission.

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Keywords: Grating; Diffraction; Negative permeability; Meta-material; Left-handed material

Meta-materials consist of artificial resonant elements, when the wavelength of incident electromagnetic wave is much larger than the dimension of the local element, the material with arrays of such elements can be homogenized. The homogenized material may have unusual EM property, such as negative permittivity and permeability [1,2]. The left-handed material (LHM) [3] is a typical meta-material currently under an intense study. Most left-handed materials [4–6] have metallic circuits as the resonant elements, as well known, the metal itself will not exhibit magnetic properties (except ferromagnetic materials), however periodically placed metallic circuits as a whole possess a negative permeability [2]. The popular resonant element to realize negative permeability is the split ring resonator (SRR) [4], which is two split loops coaxially placed together. The material with periodically placed SRRs has a negative permeability over certain frequency range, the effective negative permeability can be estimated either by analysis of the local element response [5] or by global S-parameters [7,8].

It is of interest to consider the transmission property of one meta-material layer, which is made of periodical SRR elements, this structure is similar to the grating problem widely analyzed. For TM polarized electromagnetic waves incident on a metal grating, there exist Wood anomalies [9]. The Wood anomalies are usually featured with uneven field distribution on the grating interface and with total absorption of a plane wave at certain incident angles. Fano [10] firstly associated the excitation of surface waves along the metallic gratings with such anomalies. Now, the theory that the anomalous reflectivity spectra are induced by the coupling of surface waves with the incident EM wave at the grating interface is well established. A recent application of the theory is to explain an extraordinary optical transmission through sub-wavelength hole arrays drilled on a metal film [11]. At present, it is recognized that there are the two abnormal transmission mechanisms due to a metallic grating [12]. When the grating period is smaller than the half of the incident wave wavelength, the grooves will support standing waves, leading the grating to couple with the incident plane waves; in the other case, the grating will excite coupled surface waves on the both sides of the grating [12]. So the question is: can this one layer of the SRR elements have abnormal transmissions due to grating effect under TE polarized incidence? How to characterize these transmis-

* Corresponding author at: School of Science, Beijing Institute of Technology, Beijing 100081, China. Tel./fax: +86 1986 10 68912631.
E-mail address: hugeng@bit.edu.cn (G. Hu).

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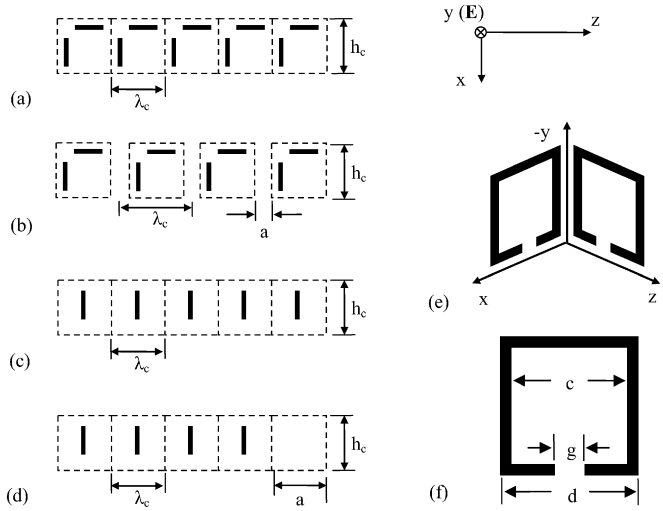

Fig. 1. The projection of SRR structures. (a) Perpendicularly arranged SRR, (b) expanded SRR, (c) vertical arranged SRR, (d) SRR lattices and a gap, (e) a unit cell, (f) the parameters of the SRR.

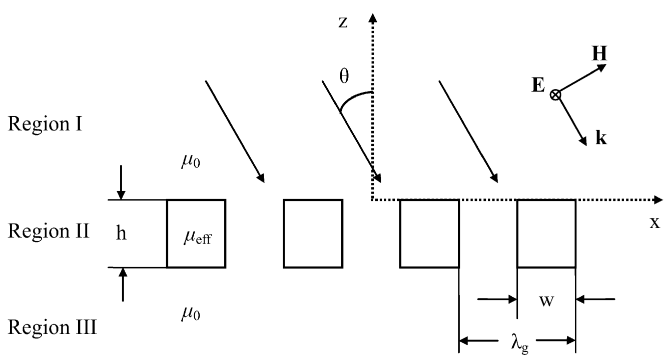

Fig. 2. Geometry for the binary grating composed of meta-materials, under the s-polarized plane wave illumination at angle θ.

sions? Answer to these questions is the objective of this Letter.

In order to proceed, we consider a layer composed of SRR elements, as shown in Fig. 1(e). It consists of two arrays of perpendicularly placed SRR elements, periodically extended in the x–z plane, the projection of this layer on the x–z plane is illustrated in Fig. 1(a) (only one array in the x-direction is shown in the figure). Another microstructure is also considered, it consists of only one array of SRR element, its projection on x–z plane is given by Fig. 1(c). For the second structure, in order to analyze the grating effect, a gap a is introduced, which is shown in Fig. 1(d). We take λ_c as the lattice constant of the meta-material, and the thickness in the propagating direction is $p \cdot h_c$, where p is an integer, representing the number of lattices in the propagation direction. The SRR is a square loop circuit with a gap, it will resonant with incident wave and generate negative permeability [13] over a certain frequency range. The microstructural parameters of the SRR are showed in Fig. 1(f). We take $d = 3.4$ mm, $c = 3.0$ mm, $g = 0.6$ mm, $\lambda_c = 5.0$ mm, $h_c = 4.5$ mm, $p = 6$. The S_{21} of the periodically arranged SRR elements in Fig. 1(a) can be obtained by HFSS, which is showed Fig. 3. A transmission stop-band is observed extending from

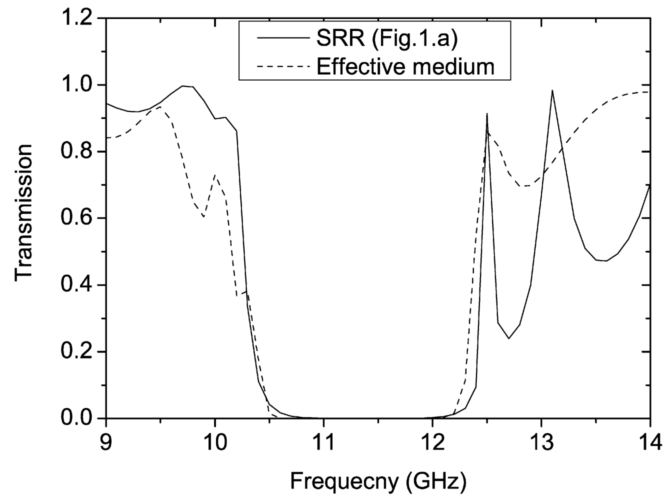

Fig. 3. Transmission properties of the SRR structure and the effective grating model.

10.5 to 12.3 GHz, this implies that the effective permeability is negative at these frequencies. By using retrieval method [8], the effective permeability can be written in the form of Lorentz model:

$$\mu_{\mathrm{eff}} = 1 - \frac{\omega_{\mathrm{mp}}^2 - \omega_{\mathrm{m0}}^2}{\omega^2 - \omega_{\mathrm{m0}}^2 + i\gamma\omega}, \tag{1}$$

where ω is the frequency of the incident wave, $\omega_{\mathrm{m0}} = 10.75$ GHz is the low-frequency limit for $\mu_{\mathrm{eff}} \leqslant 0$, and $\omega_{\mathrm{mp}} = 12.25$ GHz is "magnetic plasma frequency", $\gamma = 0.02$ GHz is the damping factor.

In order to examine the transmission property of the one layer with SRR elements over the stop-band, we consider the same structure as one-dimensional grating with negative permeability obtained from the SRR structure, as shown in Fig. 2. A gap a is introduced by adjusting the local volume fraction of the resonant element in the unit cell (Fig. 1(b)). The ridge of the grating structure is denoted by w and the grating constant by λ_g, and the thickness is h. An s-polarized wave illuminates at an angle θ from the perpendicular direction. For simplicity, the grating is assumed to be embedded in vacuum, and we establish a coordinate system with the origin on the front surface, the x-axis is along the grating vector, the z-axis is perpendicular to the grating surface. The problem is assumed infinitely extended in the y-axis direction.

Firstly, we consider the structure shown in Fig. 1(a) as a grating problem. In this case, the grating constants are: $a = 0$, $\lambda_g = w$, that means we have a uniform layer of meta-material. The transmission property of the grating is analyzed by using the RCWA [14] method, the S_{21}-parameter of the effective grating with $\lambda_g = w$ is showed in Fig. 3. It is found that over the stop-band frequency, the SRR structure and grating agree well with each other. There are some differences at frequencies lower than 10.5 GHz and higher then 12.3 GHz. When the frequency is lower than 10.5 GHz, the absolute value of the structure is beyond the maximum of the retrieval method [15]. While for the frequency higher than 12.3 GHz, the SRR is experiencing a second order resonance [5].

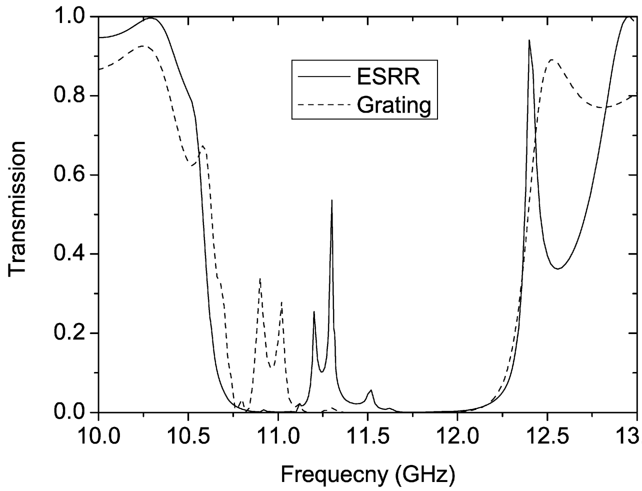

Fig. 4. Transmission of the expanded SRR structure and the effective grating, abnormal transmission peaks appears in the stop-band where the SRR have negative permeability.

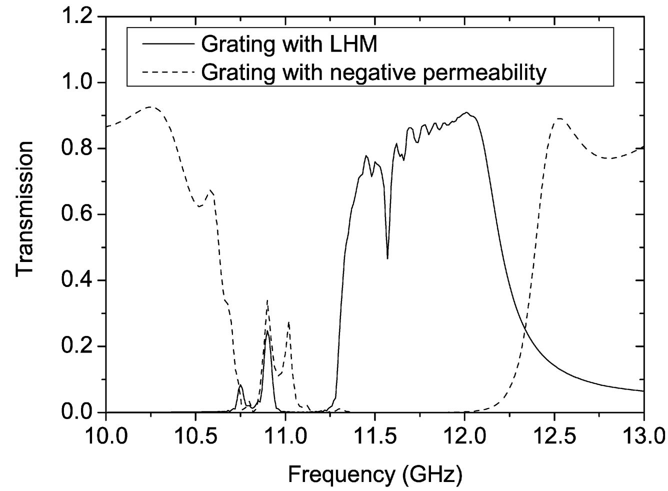

Fig. 5. Transmission of the effective grating composed of LHM, abnormal transmission peaks appears in the same frequency as the grating made from negative permeability material.

Now we set the lattice constant to be $\lambda_g = 8.0$ mm, and keep the other parameters unchanged (e.g., the local volume fraction of the resonant element in unit cell is fixed), a vacuum gap $a = 3.0$ mm is introduced. This structure is termed as the expanded SRR structure (ESRR) in the following discussion. The S_{21} of the ESRR is illustrated in Fig. 4. Compared to Fig. 3, an abnormal transmission appears in the stop-band of the negative permeability. In order to further examine this phenomenon, we consider an effective grating with constants $\lambda_g = 8.0$ mm, $w = 6.5$ mm, $h = p \cdot h_c = 27$ mm. The transmission property of the grating is also plotted in Fig. 4. It is found that the transmission curves obtained from the ESRR and the effective grating have the same form, except a small frequency shift. According to the duality, the abnormal transmission corresponds to the waveguide resonance in metallic grating [13,16–19]. If the dissipation is neglected, this transmission can be extended to 100%, which consists with the result given by Popov et al. [20]. So we have shown that the SRR structure can have simultaneously both negative permeability and grating effect, responsible for the stop-band and transmission peak respectively. It is also shown that these effects can be predicted by the effective grating structure.

When the SRR is combined with strips to form LHM [4], the grating effect still remains. Considering an homogenized LHM grating, its permittivity follows the Drude model with the plasma frequency of 17.3 GHz, and its permeability take the same value as above discussion. The grating parameters remain the same as those in Fig. 4. The transmission of the LHM grating is illustrated in Fig. 5. The broad pass-band between 11.5 GHz and 12.3 GHz implies that the material has both negative permittivity and permeability, which corresponds to the stop-band of the SRR structure only. In addition, a peak appears at about 10.9 GHz, and this is consistent with the abnormal transmission in negative permeability grating.

Now we examine the structure shown in Fig. 1(d). The structure is repeated along the x-axis with a constant $\lambda_g = 5 \times \lambda_c + a = 5 \times 4.6 + 3.0 = 26.0$ mm, which consists of five successive lattices and a gap, the whole is periodically repeated. Again we consider an effective grating with the constants: $\lambda_g = 26.0$ mm, $w = 23.0$ mm. The operating frequency is over 10.5–13 GHz, so the grating constant is comparable to the incident wave length. It is expected the s-polarized wave will excite surface wave at the surface of this structure. The surface wave vector can be estimated from Maxell equation and the boundary conditions, which leads to:

$$k_{\mathrm{spp}} = \pm k_0 \sqrt{\frac{\mu_z(\mu_x - 1)}{\mu_z \mu_x - 1}}, \tag{2}$$

where k_{spp} is the wave vector of surface wave, and $\mu_z = 1$, $\mu_x = \mu_{\mathrm{eff}}$ (retrieved from Fig. 1(c)). The surface wave will couple with s-polarized wave through a grating surface at the following condition:

$$\mathbf{k}_{\mathrm{spp}} = \mathbf{k}_0 \sin\theta \pm N\mathbf{k}_g, \tag{3}$$

where N is an integer, $\mathbf{k}_0$ is the vacuum vector, and $\mathbf{k}_g$ is the grating vector.

Fig. 6 gives the transmission curve of the above SRR structure. A transmission peak is found at 11.55 GHz, corresponding to wavelength of 26.0 mm, which is equal to the grating constant λ_g. The dashed line in Fig. 6 represents the transmission property of the effective grating with $\mu_x = \mu_{\mathrm{eff}}$ and $\mu_z = 1$. The transmission properties predicted by the two methods agree with each other, only 0.1 GHz difference is observed. This suggests that SRR structures can be used to excite surface waves under a plane wave illumination. In fact, the transmission peak at 11.55 GHz is the results of surface wave and waveguide resonance. By increasing the illuminating angle of the plane wave, the dispersion curve of the transmission peak deviates into four branches, as shown in Fig. 7. From Eq. (3), the surface wave is highly dependent on the incident angle, however the waveguide resonance is independent, these are clearly distinguished in Fig. 7.

To conclude, we have analyzed the transmission properties of the periodical SRR structure, and proposed a grating model

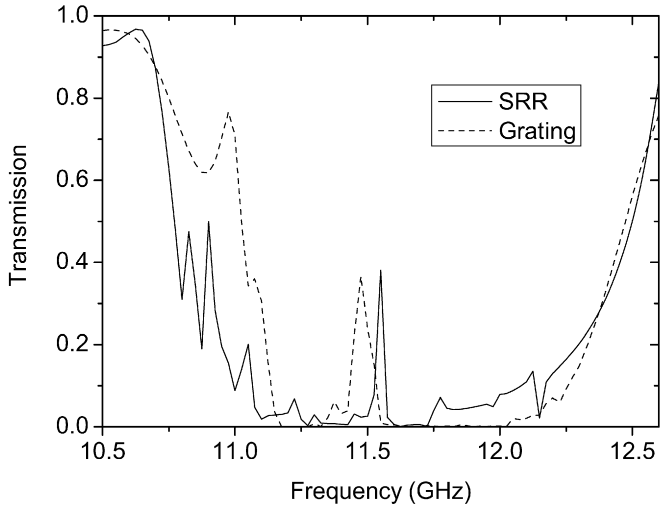

Fig. 6. Transmission of the SRR/gap structure and the effective grating with constant comparable to the operating wavelength. Abnormal transmission is found in the stop-band of negative permeability.

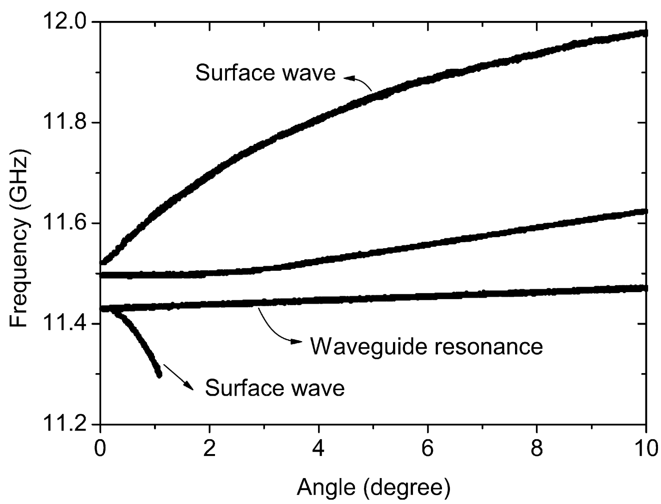

Fig. 7. The dispersive relation of the surface wave.

to predict the abnormal transmission in the stop-band. The results show that at certain condition, a transmission peak can take place over the stop-band of the SRR structure. This transmission is due to the grating effect through wave guide mode or surface mode. The proposed grating model with meta-material can predict this abnormal transmission correctly.

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