Pergamon

Mechanics Research Communications, Vol. 24, No. 1, pp. 65-68, 1997

0093-6413/97 $17.00 + .00

PII S0093-6413(96)00079-1

MICROMECHANICAL ANALYSIS OF FATIGUE PROPERTIES OF METAL-MATRIX COMPOSITES

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(Received 9 November 1995; accepted for print 2 August 1996)

Introduction

An incremental micromechanical theory is necessary for modeling the cyclic properties of metal matrix composites. Lin et al. [1] proposed an incremental theory based on Eshelby inclusion transformation theory[2]. Wakasima[3] considered the matrix plastic strain as a source of eigenstrain occurred in the inclusions; the constraint of the matrix on the inclusion is always considered elastic. He formulated an incremental theory to model the cyclic properties of metal matrix composites. In this paper an incremental micromechanical method will be proposed in the context of anisotropic elasticity along the line proposed by Hill[4], and the interaction between the inclusions will be accounted for with the help of Mori-Tanaka mean field theory[5]. The method will be applied for predicting the tensile, cyclic properties and fatigue life of metal matrix composites.

Theoretical analysis

Composite compliance tensor

The Mori-Tanaka mean field theory[5] will be applied in this paper to determine the elastic composite stiffness tensor. The studied composite is assumed to consist of two phases: the matrix and aligned isotropic inclusions, C_I and C_m denote the stiffness tensors respectively, f is the volume fraction of the inclusions. Taking a representative volume of such composite, a uniform stress Σ is applied along its boundary. It can be shown that the average matrix stress and the composite strain are related to the applied stress by[3]

$$\sigma_m = K\Sigma, \qquad E = M_{eff}\Sigma \tag{1}$$

where $K = I - C_m(I+Q)^{-1}QC_m^{-1}$, $Q = f(S-I)[(C_m - C_I)S - C_m]^{-1}(C_I - C_m)$, and S is Eshelby tensor. M_{eff} is the composite effective compliance tensor and its expression is

$$M_{eff} = [I + (S-I)^{-1}(I+Q)^{-1}Q]C_m^{-1} \tag{2}$$

In the incremental loading case, equation (1) can be written in the incremental form

$$\Delta\sigma_m = K\Delta\Sigma \qquad \Delta E = M_{eff}\Delta\Sigma \tag{3}$$

In this case, the matrix moduli must be taken as the tangent moduli on which the Eshelby tensor depends. When the matrix undergoes a plastic deformation, the above incremental relations will be applied and M_{eff} and K will depend on the plastic state of the matrix through its tangent moduli.

Eshelby tensor

For an ellipsoidal inclusion embedded in an isotropic matrix, the Eshelby tensor has a simple analytical form (the readers can refer the monograph by Mura[6]). However when the matrix has a plastic deformation, the relation between its incremental stress and strain can not be arranged into a simple form as in the elastic isotropic case. The Eshelby tensor for an anisotropic media has not simple analytical expression and it must be evaluated numerically. Assuming the matrix obeys the mixed hardening law, its yield function is:

$$F = \sqrt{\frac{3}{2}(s_{ij} - c\varepsilon_{ij}^p)(s_{ij} - c\varepsilon_{ij}^p)} - \sigma_{y_0} - \varphi(\tilde{\varepsilon}_p) = 0 \tag{4}$$

where c is a constant characterizing kinetic hardening, s_{ij} is the deviatoric part of the stress tensor, ε_{ij}^p is plastic strain and $\tilde{\varepsilon}_p$ is the usual equivalent plastic strain.

With the aid of the consistency condition and the normality flow rule, the matrix tangent compliance tensor can be expressed as

$$M_{ijkl}^t = M_{ijkl}^e + \frac{3}{2I}\frac{1}{2\varphi' + 3c}(s_{ij} - c\varepsilon_{ij}^p)(s_{kl} - c\varepsilon_{kl}^p) \tag{5}$$

where M_{ijkl}^e is the matrix elastic compliance tensor for an isotropic material, $I = \frac{1}{2}(s_{ij} - c\varepsilon_{ij}^p)(s_{ij} - c\varepsilon_{ij}^p)$ and $\varphi' = d\varphi / d\tilde{\varepsilon}_p$. When $\varphi' = 0$ or $c = 0$, the previous hardening law reduces to the pure kinetic or isotropic hardening.

The Eshelby tensor for an ellipsoidal inclusion in an anisotropic media characterized by the compliance tensor M_{ijkl}^t has been theoretically analyzed by Mura[6]. It can be expressed by [7]

$$S_{ijkl} = P_{ijpq}C_{pqkl} \tag{6}$$

and $P_{ijpq} = \dfrac{1}{16\pi}\displaystyle\int_{-1}^{1} dx \int_0^{2\pi} d\phi (A_{ip}^{-1}K_qK_j + A_{jq}^{-1}K_pK_i + A_{jp}^{-1}K_qK_i + A_{iq}^{-1}K_pK_j)$

with $K_1 = (1-x^2)^{\frac{1}{2}}\cos\phi$, $K_2 = \frac{a}{b}(1-x^2)^{\frac{1}{2}}\sin\phi$, $K_3 = \frac{a}{c}x$, $A_{ij} = A_{ji} = C_{ipjq}K_pK_q$

a, b, c are the half axes of the ellipsoidal inclusion. Tensor C_{pqkl} is the inverse of the tensor M_{ijkl}^t

P_{ijpq} can be numerally evaluated by Gauss method, so the Eshelby tensor S used in the section 2.1 for the each incremental step is then obtained.

Numerical applications

When the matrix enters the plastic deformation range, the external load must be realized in an incremental manner. In each incremental step, the matrix tangent compliance tensor is determined by equation (5) and the Eshelby tensor are obtained through equation (6). We get the composite incremental stress and strain by equations (2,3). This process is repeated until the required load or cycle is reached. In the elastic case or unloading, the matrix elastic compliance tensor M^{e}_{ijkl} must be used.

Taking the Al/SiC composite as an example, two kinds of matrix are examined: one is Al+3.5%Cu[8], the other is Aluminum A356[9]. Their materials constant are

TABLE 1: Material constants for the matrices

	E_0 (GPa)	ν_0	σ_{y0} (MPa)	h (MPa)	n	c (MPa)
Al+3.5%Cu	70	0.33	175	90	0.25	2500
Al A356-T6	70	0.33	210	75	0.25	2000

The material constant for the SiC are[8]: $E_f = 450GPa$, $\nu_f = 0.17$.

Figure 1 illustrates the predicted tensile and cyclic hardening behavior (solid line) for the particulate reinforced composite(Al+3.5%Cu, f=13%), together with the experimental data (dash line)[8]. In the case of cyclic loading, the applied strain is controlled, the corresponding stress is taken as the maximum stress after 15 cycles. We note that the proposed method can give good predictions for both tensile and cyclic hardening behavior of the composite.

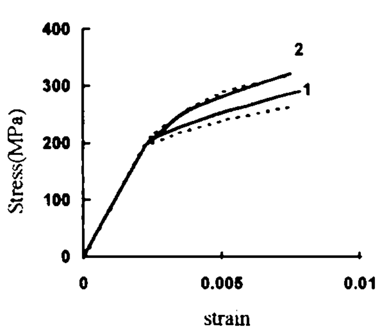

FIG.1 Comparison between the predicted experimental tensile and cyclic hardening curves of the composite (1: tensile; 2: cyclic)

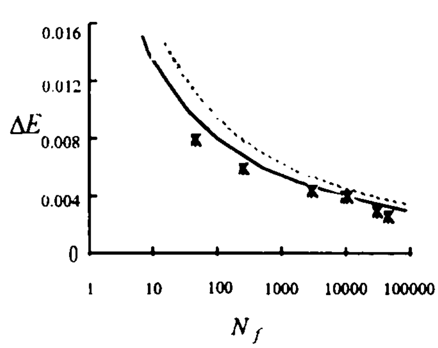

FIG. 2 Comparison between the predicted (solid line) and measured(•) fatigue life for the composite;

In this paper, the composite fatigue life is assumed to be controlled by the initiation of a crack in the matrix material, as in references[9]. The matrix average quantities will be used for the composite fatigue life prediction. The matrix fatigue life and the applied strain range in the matrix is related by the Coffin-Manson relation[10].

$$\frac{\Delta\varepsilon_m}{2} = \frac{\sigma'_f}{E_0}(2N_f)^b + \varepsilon'_f(2N_f)^c \tag{7}$$

where $\Delta\varepsilon_m$ is the maximum principle strain range in the matrix, E_0 is the matrix elastic Young's modulus. σ'_f, ε'_f, x and y are the matrix material constants. For the composite under the cyclic loading (the applied composite strain range is ΔE), the micromechanical model allows one to determine the maximum average strain range in the matrix $\Delta\varepsilon_m$. With the help of equation (7), the cycle to failure for the composite under the applied strain range ΔE can then be determined.

This method will be applied to a particulate reinforced Al/SiC composite (Al A326-T6), the volume fraction of the SiCp is 20%. $x = -0.119$, $y = -0.544$, $\sigma'_f = 502MPa$, $\varepsilon'_f = 0.0116$ as determined by Ogarevic[9]. Figure 2 shows the predicted fatigue life for the composite (solid line), together with the simulation of the matrix fatigue life (dash line) and the experimental results[9], the proposed method gives also a reasonable prediction of fatigue life for the composite.

Conclusions

In this paper, an incremental method is proposed along the line given by Hill[4]. It is then applied to predict the cyclic behavior of metal-matrix composites. In each incremental step, the matrix is considered to be an anisotropic material whose stiffness tensor is chosen as the tangent moduli of the studied matrix. With the aid of Mori-Tanaka mean field theory and numerical solution of the Eshelby tensor, the incremental stress and strain relation of the composite is derived. The predictions of the tensile, cyclic hardening behavior and the fatigue life for a particulate reinforced composite agree quantitatively well with the ones given in the literature.

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