Acta Mechanica 187, 139–149 (2006)
DOI 10.1007/s00707-006-0319-6

Acta Mechanica
Printed in Austria

Micromechanical modeling of local field distribution for a planar composite under plastic deformation

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Received December 16, 2005; revised February 27, 2006
Published online: May 24, 2006

Summary. Due to statistical distribution of local material property, local stress and strain fields in a composite are random in nature. Classical micromechanical methods can only predict the average value of these local fields in different phases. An analytical method, which combines the maximum entropy theory and secant moduli method, is proposed in this paper. The distribution of the local field for a planar composite with plastic deformation is examined by the proposed method. The results show that with increase of plastic deformation the stress field in the matrix becomes more and more inhomogeneous. The predicted results on the stress distribution are in reasonable agreement with finite element simulation. Some salient features near the elastic and plastic deformation transition revealed by finite element simulation are also discussed.

1 Introduction

The overall elastic property of composite material can be well predicted by micromechanical methods, developed in the past few decades (Milton [1], Torquato [2], Nemat-Nasser and Hori [3]). The idea of these methods is to define a representative volume element (RVE) under uniform loading conditions, and then to find the average stress and strain in each phase. The relation between the average stress and strain in the RVE provides the effective constitutive relation for the composite material. Such homogenization technique is further developed for predicting the nonlinear behavior of composites. For examples Ponte Castañeda proposed a variational method [4], Qiu and Weng [5] and Hu [6] proposed a secant moduli method. Normally the average stress and strain in the phases can be predicted with reasonable accuracy with these micromechanical methods. However due to the complex morphology of the composite, especially the continuous nature for the matrix phase, the local field distributions are extremely complicated. The information on these local stress and strain distributions is important to assess the early plastic deformation and damage of composite materials. However this kind of information can not be provided by the classical micromechanical methods. The objective of this paper is to propose one, which combines the classical micromechanical method and information entropy theory. This method will be shown to be able to predict local stress and strain distributions in composite materials under plastic deformation.

The paper will be arranged as follows: some general relations of the micromechanical method will be presented in Sect. 2; in Sect. 3, the information entropy theory will be applied to predict the elastic local field distribution; the local field distribution during the plastic deformation will be examined by the secant moduli method, which will be explained in Sect. 4; some numerical applications of the proposed method and comparison with the finite element method will be presented in Sect. 5.

2 General relations and overall elastic properties for composite materials

Consider a two-phase planar composite, where $\mathbf{C}_0(\mu_0, k_0)$, $\mathbf{C}_1(\mu_1, k_1)$ denote the moduli (shear modulus, planar bulk modulus for two-dimensional isotropic material) for the matrix and the reinforced phase, respectively. We consider only a two-dimensional problem. The reinforced phase is assumed to be cylindrical in shape, and its volume fraction is denoted by f. Taking a representative volume element (RVE) of such a composite under an affine displacement $(\mathbf{u} = \bar{\boldsymbol{\varepsilon}} \cdot \mathbf{x})$ or uniform traction $(\boldsymbol{\sigma} \cdot \mathbf{n} = \bar{\boldsymbol{\sigma}} \cdot \mathbf{n})$ boundary condition, it is

$$\langle \boldsymbol{\varepsilon} \rangle = \bar{\boldsymbol{\varepsilon}}, \quad \langle \boldsymbol{\sigma} \rangle = \bar{\boldsymbol{\sigma}}, \tag{1}$$

where $\bar{\boldsymbol{\varepsilon}}$ and $\bar{\boldsymbol{\sigma}}$ are called macroscopic strain and stress tensors, $\mathbf{x}$ and $\mathbf{n}$ are position and unit outward normal vectors, $\boldsymbol{\varepsilon}$ and $\boldsymbol{\sigma}$ are the local strain and stress tensor in the RVE. $\langle \bullet \rangle \equiv \frac{1}{V}\int_V \bullet \, dV$ is defined as the volume average of the said quantity. When the ergodic assumption is adopted, the ensemble average equals the volume average, that is:

$$\langle A \rangle = \frac{1}{V}\int_V A dV = \int A p(A) dA, \tag{2}$$

where $p(A)$ is the probability density of random variable A.

Here we can define the effective stiffness tensor $\overline{\mathbf{C}}$ and the compliance tensor $\overline{\mathbf{M}}$ for the composite by:

$$\bar{\boldsymbol{\sigma}} = \overline{\mathbf{C}} : \bar{\boldsymbol{\varepsilon}}, \quad \bar{\boldsymbol{\varepsilon}} = \overline{\mathbf{M}} : \bar{\boldsymbol{\sigma}}. \tag{3}$$

Another important formula in micromechanics is Hill's relation, which states that if there is only uniform traction or only affine displacement condition on the RVE's boundary, the average internal energy is related to the macroscopic quantity by:

$$\langle \boldsymbol{\sigma} : \boldsymbol{\varepsilon} \rangle = \bar{\boldsymbol{\sigma}} : \bar{\boldsymbol{\varepsilon}}. \tag{4}$$

Equation (4) can also be further written in the following form with help of the local and global elastic relations:

$$\langle \boldsymbol{\varepsilon} : \mathbf{C} : \boldsymbol{\varepsilon} \rangle = \bar{\boldsymbol{\varepsilon}} : \overline{\mathbf{C}} : \bar{\boldsymbol{\varepsilon}}, \quad \langle \boldsymbol{\sigma} : \mathbf{M} : \boldsymbol{\sigma} \rangle = \bar{\boldsymbol{\sigma}} : \overline{\mathbf{M}} : \bar{\boldsymbol{\sigma}}, \tag{5}$$

where $\mathbf{C}$ and $\mathbf{M}$ are the local stiffness and compliance tensors of the composite, respectively.

Equations (1) and (3)–(5) hold for any composite, they also provide constraints for permissible local fields, such as local stress and strain fields. $\overline{\mathbf{C}}$ and $\overline{\mathbf{M}}$ depend on the phase property and the detailed microstructure. There are many micromechanical models which can estimate these effective properties, i.e., the Mori-Tanaka model, self-consistent model, double inclusion model, etc. (see for example Hu and Weng [7] for more details). Here in this paper, for simplicity, Mori-Tanaka mean field theory will be used to estimate the overall elastic property of the composite. In this case the effective elastic compliance of the composite reads [6]

$$\overline{\mathbf{M}} = \mathbf{M}_0 + f[(\mathbf{M}_1 : \mathbf{M}_0^{-1} - \mathbf{I})^{-1} + (1-f)(\mathbf{I} - \mathbf{S})]^{-1} : \mathbf{M}_0, \tag{6}$$

where $\mathbf{M}_0$ and $\mathbf{M}_1$ denote the compliance tensors of the matrix and randomly dispersed phase, separately. $\mathbf{I}$ is the fourth order unit tensor, and $\mathbf{S}$ is the Eshelby tensor (see Mura [8] for the details).

3 Information entropy approach for local field distribution

As discussed in the previous section, the objective of this paper is to determine the local stress and strain distributions inside of the composite, however classical micromechanics can only deliver the expectation values of these local fields in each phase. In this paper, the overall elastic property of the composite is estimated by Mori-Tanaka's method, and it is given by Eq. (6). This property is assumed to be known in the following analysis. All we know is that the local fields must satisfy the constraints given by Eqs. (1) and (3)–(6). There are obviously many distributions of the internal fields coherent with the previous constraints. So we have to figure out among the possible distributions the most suitable one. To this end, an additional principle will be needed. The information entropy approach proposed by Kreher and Pompe [9] will be used, and will be explained briefly in the following.

Information entropy is the measure of amount of *uncertainty* or *missing information* in a probability distribution. To avoid additional assumptions, one has to choose the probability distribution which maximizes the information entropy. For our problem, the local material property is a random field variable, which leads to the local field to be a random field, too. What we are interested in is the joint probability distribution of the local strain $\boldsymbol{\varepsilon}$ and the local material property $\mathbf{C}$, $p = p(\boldsymbol{\varepsilon}, \mathbf{C})$. Information entropy is defined as [9]

$$S_I = -\iint p(\boldsymbol{\varepsilon}, \mathbf{C}) \ln \frac{p(\boldsymbol{\varepsilon}, \mathbf{C})}{p_0} d\boldsymbol{\varepsilon} d\mathbf{C} = \left\langle -\ln \frac{p(\boldsymbol{\varepsilon}, \mathbf{C})}{p_0} \right\rangle, \tag{7}$$

where p_0 is an arbitrary reference value ensuring a positive value for S_I. Obviously the maximum of S_I must be achieved under some constraints of $p(\boldsymbol{\varepsilon}, \mathbf{C})$, expressed in terms of the local strains by

$$\langle \boldsymbol{\varepsilon} \rangle = \bar{\boldsymbol{\varepsilon}}, \quad \langle \mathbf{C} : \boldsymbol{\varepsilon} \rangle = \overline{\mathbf{C}} : \bar{\boldsymbol{\varepsilon}}, \quad \langle \boldsymbol{\varepsilon} : \mathbf{C} : \boldsymbol{\varepsilon} \rangle = \bar{\boldsymbol{\varepsilon}} : \overline{\mathbf{C}} : \bar{\boldsymbol{\varepsilon}}. \tag{8}$$

Another constraint comes from the assumption of the probability distribution for the local material $p^C(\mathbf{C})$, which is expressed to be

$$\int p(\boldsymbol{\varepsilon}, \mathbf{C}) d\boldsymbol{\varepsilon} = p^C(\mathbf{C}). \tag{9}$$

Thus the determination of $p(\boldsymbol{\varepsilon}, \mathbf{C})$ can be achieved by the stationary condition for the following variational problem:

$$\delta S_I = 0 \tag{10}$$

under the constraints given by Eqs. (8) and (9).

Since the effective constant prescribes the elastic energy of the system, the maximum information entropy method is analogous to statistical mechanics. Although lacking in all the detailed information on microstructure, one can still determine reasonable fluctuation on internal fields. We note that the overall quantities $\bar{\boldsymbol{\varepsilon}}$, $\bar{\boldsymbol{\sigma}}$, $\overline{\mathbf{C}}$ and the local property distribution $p^C(\mathbf{C})$ are considered as input. The stationary condition (10) can be solved by a standard technique. Finally we get the Gaussian probability distributions for the local strain field whose mean value and standard deviation are functions of the local material and the known macroscopic quantities. The general solution is [9]:

$$p(\boldsymbol{\varepsilon}, \mathbf{C}) = p^C(\mathbf{C}) p^{\varepsilon}(\boldsymbol{\varepsilon}|\mathbf{C}), \tag{11}$$

$$p^{\varepsilon}(\boldsymbol{\varepsilon}|\mathbf{C}) = \frac{1}{Z} \exp\left[-\frac{1}{2d}(\boldsymbol{\varepsilon} - \boldsymbol{\gamma}) : \mathbf{K}^{-1} : (\boldsymbol{\varepsilon} - \boldsymbol{\gamma})\right], \tag{12}$$

where $p^{\varepsilon}(\boldsymbol{\varepsilon}|\mathbf{C})$ means the conditional probability density of $\boldsymbol{\varepsilon}$ when $\mathbf{C}$ is given.

$Z = \iint \exp\left[-\frac{1}{2d}(\boldsymbol{\varepsilon} - \boldsymbol{\gamma}) : \mathbf{K}^{-1} : (\boldsymbol{\varepsilon} - \boldsymbol{\gamma})\right] d\boldsymbol{\varepsilon}$, and

$$\boldsymbol{\gamma}(\mathbf{C}) = \mathbf{C}^{-1} : \boldsymbol{\eta}_{\varepsilon} + \boldsymbol{\eta}_{\sigma} = \langle \boldsymbol{\varepsilon}|\mathbf{C} \rangle, \tag{13}$$

$$\mathbf{K} = \mathbf{C}^{-1}/2. \tag{14}$$

Here $\boldsymbol{\eta}_{\varepsilon}$, $\boldsymbol{\eta}_{\sigma}$ and d are completely determined by the phase and the overall properties, as well as the applied loads, they are:

$$\boldsymbol{\eta}_{\varepsilon} = \mathbf{C}^{-} : (\mathbf{C}^{+} - \mathbf{C}^{-})^{-1} : (\mathbf{C}^{+} - \overline{\mathbf{C}}) : \bar{\boldsymbol{\varepsilon}}, \tag{15.1}$$

$$\boldsymbol{\eta}_{\sigma} = (\mathbf{C}^{+} - \mathbf{C}^{-})^{-1} : (\overline{\mathbf{C}} - \mathbf{C}^{-}) : \bar{\boldsymbol{\varepsilon}}, \tag{15.2}$$

$$d = \frac{1}{3} \bar{\boldsymbol{\varepsilon}} : (\mathbf{C}^{+} - \overline{\mathbf{C}}) : (\mathbf{C}^{+} - \mathbf{C}^{-})^{-1} : (\overline{\mathbf{C}} - \mathbf{C}^{-}) : \bar{\boldsymbol{\varepsilon}}, \tag{15.3}$$

where $\mathbf{C}^{+} = \langle \mathbf{C} \rangle$, $\mathbf{C}^{-} = \langle \mathbf{C}^{-1} \rangle^{-1}$ are Voigt and Reuss bounds for the composite, respectively.

Once the above quantities are determined, the average value and variant of the random stress field corresponding to a certain given local elastic tensor $\mathbf{C}$ can be readily derived as

$$\bar{\boldsymbol{\sigma}}_C = \langle \boldsymbol{\sigma}|\mathbf{C} \rangle = \boldsymbol{\eta}_{\varepsilon} + \mathbf{C} : \boldsymbol{\eta}_{\sigma}, \tag{16}$$

$$\langle (\boldsymbol{\sigma} - \bar{\boldsymbol{\sigma}}_C) \otimes (\boldsymbol{\sigma} - \bar{\boldsymbol{\sigma}}_C)|\mathbf{C} \rangle = \frac{d}{2}\mathbf{C}. \tag{17}$$

4 Evolution of internal field distribution during plastic deformation

In the previous section, the random local elastic stress field distribution has been determined (by Eqs. (11)–(17)) using the maximum entropy theory. Up to now all the results are applied for elastic composites. In the following, we will make use of these results to the elasto-plastic composite and try to derive the change of the probability distribution of the random internal fields during plastic deformation. To this end, the secant moduli method [5], [6] will be used. We suppose that the matrix material is an elasto-plastic material and the reinforced phase is a pure elastic material. The plastic deformation of the matrix is characterized by the following strain potential:

$$\psi = \varphi(\boldsymbol{\sigma}_{eff}) + \frac{1}{2K_0}\boldsymbol{\sigma}^2, \tag{18}$$

where $\boldsymbol{\sigma} = \mathrm{tr}(\boldsymbol{\sigma})/3$ is hydrostatic stress, $\boldsymbol{\sigma}_{eff} = (3\mathbf{s} : \mathbf{s}/2)^{1/2}$ is von Mises effective stress and K_0 is the 3D bulk modulus. When the matrix enters a plastic state, with $\boldsymbol{\sigma}_{eff}$ exceeding the initial yield stress of the matrix $\boldsymbol{\sigma}_Y$, its secant shear modulus μ_0^s is given by

$$\mu_0^s = \frac{\boldsymbol{\sigma}_{eff}}{3\varphi'(\boldsymbol{\sigma}_{eff})}, \tag{19.1}$$

since the planar bulk modulus is $k_0 = K_0 + \frac{1}{3}\mu_0$, and K_0 remains unchanged during plastic deformation due to the plastic incompressibility. The matrix's secant planar bulk modulus k_0^s is given by

$$k_0^s = K_0 + \frac{1}{3}\mu_0^s. \tag{19.2}$$

With such defined secant moduli for the matrix material, the secant modulus method can be explained as the following: for any given macroscopic load $\overline{\boldsymbol{\sigma}}$, at which the matrix has entered a plastic state, for a given average effective stress of the matrix $\langle \boldsymbol{\sigma}_{eff}|\mathbf{C}\rangle(>\boldsymbol{\sigma}_Y)$, the secant moduli of the matrix can be evaluated by Eq. (19). The stiffness tensors $\overline{\mathbf{C}}^s$ ($\overline{\mathbf{M}}^s$, compliance tensor) of a linearized elastic composite with an elastic matrix having an average secant modulus of the actual matrix can be determined from Eq. (6), and these moduli are interpreted as the secant moduli of the actual nonlinear composite. In order to determine the change of the matrix secant moduli as function of the applied macroscopic load, the method based on second order stress moment of matrix will be used [5], [6]. The average effective stress of the matrix for the linearized composite can be written as

$$\langle \boldsymbol{\sigma}_{eff}\rangle^2 \cong \langle \boldsymbol{\sigma}_{eff}^2\rangle_0 = \frac{3}{2}\langle \mathbf{s}:\mathbf{s}\rangle_0 = \overline{\boldsymbol{\sigma}} : \left(-\frac{3\mu_0^{s^2}}{(1-f)}\frac{\partial \overline{\mathbf{M}}^s}{\partial \mu_0^s}\right) : \overline{\boldsymbol{\sigma}}. \tag{20}$$

By repeating $\langle \boldsymbol{\sigma}_{eff}|\mathbf{C}\rangle$, the nonlinear relation between the stress and strain of the composite can then be established. We apply the previous information entropy theory for each successive linear comparison composite, so the evolution of the local stress distribution can then be determined. In the following, we will apply this method to determine the local stress field variation during plastic deformation, and compare the results with finite element simulation.

5 Numerical applications

5.1 Analytical method

Here a two-phase planar composite (plane strain condition) with an elasto-plastic matrix and elastic circular inclusions is considered. The local materials are isotropic. The macroscopic load is supposed to be uniaxial tension in y-direction.

Due to the isotropy of the overall stiffness tensor, the effective shear modulus $\overline{\mu}$ and planar bulk modulus $\overline{k}$, estimated by Mori-Tanaka method, are expressed as

$$\overline{\mu} = \mu_0\left(1 + \frac{f}{2(1-f)\alpha + \mu_0/(\mu_1-\mu_0)}\right), \tag{22.1}$$

$$\overline{k} = k_0\left(1 + \frac{f}{(1-f)\beta/2 + k_0/(k_1-k_0)}\right), \tag{22.2}$$

where α and β are the two independent components of the isotropic Eshelby tensor:

$$\alpha = \frac{k_0+2\mu_0}{4(k_0+\mu_0)}, \quad \beta = \frac{2k_0}{k_0+\mu_0}. \tag{23}$$

Equation (19) is further simplified, due to the overall isotropy, as

$$\boldsymbol{\eta}_{\boldsymbol{\varepsilon}} = (1-q_k)\overline{\sigma}\mathbf{I} + (1-q_\mu)\overline{\mathbf{s}}, \tag{24.1}$$

$$\boldsymbol{\eta}_{\boldsymbol{\sigma}} = \frac{1}{3k^+} q_k \overline{\boldsymbol{\sigma}} \mathbf{I} + \frac{1}{2\mu^+} q_\mu \overline{\mathbf{s}}, \tag{24.2}$$

$$d = \frac{1}{3} \frac{k^+ - k^-}{k^+ k^-} q_k (1 - q_k) \overline{\boldsymbol{\sigma}}^2 + \frac{1}{6} \frac{\mu^+ - \mu^-}{\mu^+ \mu^-} q_\mu (1 - q_\mu) \overline{\mathbf{s}} : \overline{\mathbf{s}}, \tag{24.3}$$

where $k^+ = \langle k \rangle$, $k^- = \langle k^{-1} \rangle^{-1}$, $\mu^+ = \langle \mu \rangle$, $\mu^- = \langle \mu^{-1} \rangle^{-1}$ and $q_\mu = (1/\mu^- - 1/\overline{\mu})/(1/\mu^- - 1/\mu^+)$, $q_k = (1/k^- - 1/\overline{k})/(1/k^- - 1/k^+)$. $\overline{\boldsymbol{\sigma}}, \overline{\mathbf{s}}$ are the hydrostatic and deviatoric parts of the tensor $\overline{\boldsymbol{\sigma}}$, respectively.

The matrix is assumed to have a power law type hardening which can be expressed by $\boldsymbol{\sigma}_{eff} = \boldsymbol{\sigma}_Y + h\boldsymbol{\varepsilon}_{ep}^n$. Accordingly the strain potential of the matrix is written as

$$\psi = \frac{\boldsymbol{\sigma}_{eff}^2}{6\mu_0} + \frac{n}{n+1} \frac{1}{h^{1/n}} (\boldsymbol{\sigma}_{eff} - \boldsymbol{\sigma}_Y)^{\frac{n+1}{n}} + \frac{1}{2k_0^s} \boldsymbol{\sigma}^2, \tag{25}$$

where h and n are the plastic material constants, and $\boldsymbol{\varepsilon}_{ep}$ is the von Mises effective plastic strain. Then the secant moduli of the matrix material are given by

$$\mu_0^s = \frac{1}{(1/\mu_0) + 3[(\boldsymbol{\sigma}_{eff} - \boldsymbol{\sigma}_y)/h]^{1/n} / \boldsymbol{\sigma}_{eff}}, \quad k_0^s = K_0 + \mu_0^s/3. \tag{26}$$

With the secant moduli method discussed previously, the overall nonlinear behavior and internal field probability distribution of the considered composite system at any instant can be calculated. The material constants in the computation are taken as: $k_0 = 75.5$ GPa, $k_1 = 135$ GPa, $\mu_0 = 25.7$ GPa, $\mu_1 = 89.3$ GPa, $\boldsymbol{\sigma}_Y = 250$ MPa, $h = 600$ MPa, $n = 0.455$, and the volume fraction of the inclusion is $f = 0.2$.

5.2 FEM calculation

Finite Element calculation for the same composite is also performed, which is shown in Fig. 1. There are 60 circular inclusions randomly scattered within the matrix, whose radius is adjusted to satisfy the volume fraction of the inclusion $f = 0.2$. All the material parameters are taken to be the same as those in the analytical model. The sample was stretched by a uniform displacement on the boundary in the y-direction, and in the x-direction, the periodic condition is prescribed.

The local stress fields and the probability distribution of various stress components as well as the overall nonlinear behavior of the composite can then be analyzed numerically.

5.3 Comparison results

The overall stress-strain curves of the composite, estimated by the previous two models are shown in Fig. 2. It is seen that there is a good agreement between these two methods on the overall nonlinear stress and strain relation. There is no sharp yielding point in the curve predicted by FEM calculation due to the more precise consideration for the local yielding happening in the matrix material.

The probability distributions of three stress components $\boldsymbol{\sigma}_y, \boldsymbol{\sigma}_x, \tau_{xy}$ in the matrix at different loading levels are computed with the analytical method proposed in the previous sections, they are also compared with the Finite Element calculation. The results are illustrated in Figs. 3, 4 and 5, respectively.

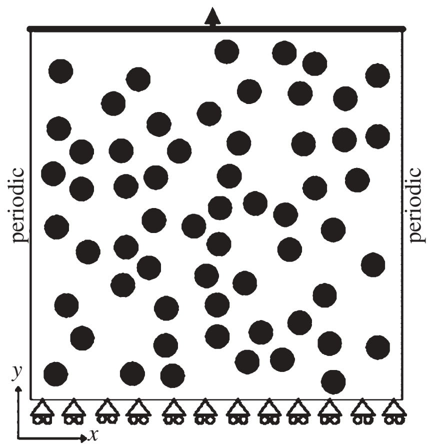

Fig. 1. Finite element model of a RVE

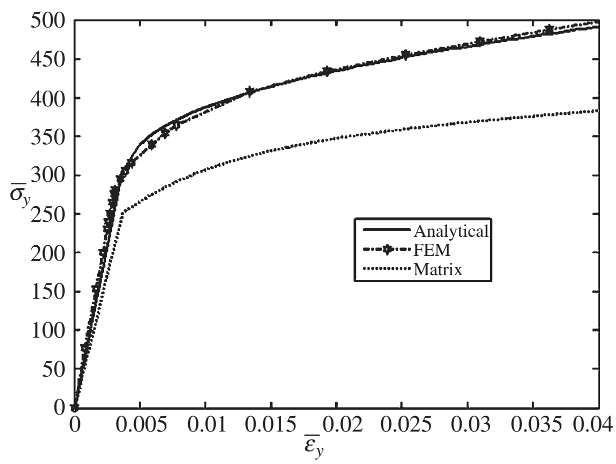

Fig. 2. Comparison between analytical and FEM methods on overall stress-strain relation

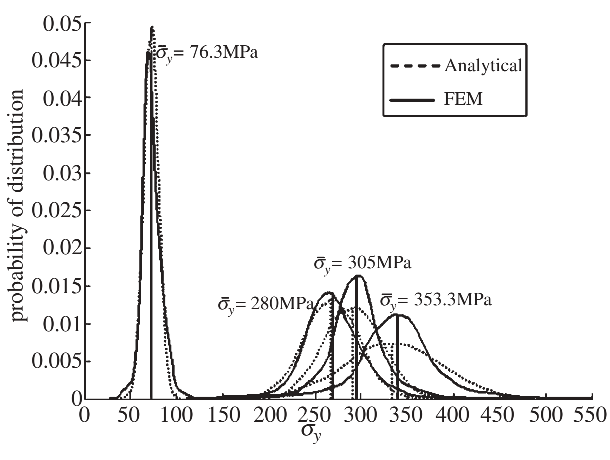

Fig. 3. Probability distribution of σ_y at different loading levels

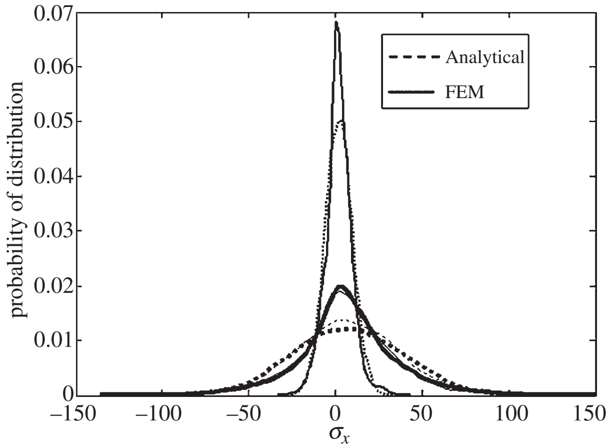

Fig. 4. Probability distribution of σ_x at different loading levels

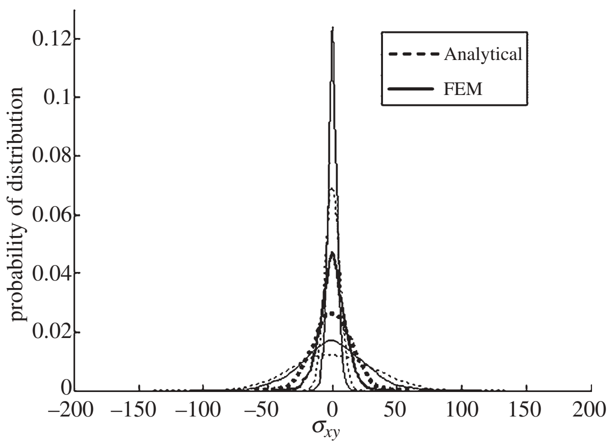

Fig. 5. Probability distribution of σ_{xy} at different loading levels

With the increase of loading, the yielding and hardening phenomena take place in the matrix in a non-uniform manner. Both the proposed method and FEM calculation show that the distributions of the local stress fields becomes more and more smooth and flat. This indicates that the scatter of these local fields becomes more and more important during plastic deformation. The distributions for $\boldsymbol{\sigma}_x, \tau_{xy}$ are around zero, as expected for a uniaxial loading $\overline{\boldsymbol{\sigma}}_y$. It is also seen that with an increase of loading the predictions by the analytical method and Finite Element calculation show more discrepancy. The analytical method predicts more important stress variation in the matrix than that by FEM simulation, probably due to the simplification made by the secant moduli method. However, the proposed analytical method can indeed capture the major change of the local field distribution in the composite under plastic deformation.

We also note that near the yielding point of the matrix material the stress distribution ($\boldsymbol{\sigma}_y$) obtained from FEM simulation shows an abnormal change. For example, when the macroscopic stress varies from 280 MPa to 305 MPa, the variant of the stress distribution decreases (see Fig. 3). This is because there is a substantial rearrangement in the internal stress field when the local stress reaches the yielding threshold of the matrix material. This issue will be examined in detail later. The proposed method based on the secant moduli method cannot consider such phenomena.

From the probability distribution of the stress components, we can evaluate the distribution $\rho(\boldsymbol{\sigma}_{eff})$ of the von Mises effective stress. Analytical determination of $\rho(\boldsymbol{\sigma}_{eff})$ needs a high dimensional infinite integration. Here as a convenient alternative, Monte-Carlo method has been used to obtain $\rho(\boldsymbol{\sigma}_{eff})$. The distributions of the von Mises stress at different loading levels are shown in Fig. 6. Obviously the distribution of the von Mises stress is no longer of Gaussian type because of the non-negativity of $\boldsymbol{\sigma}_{eff}$. The evolution of the probability distribution of $\boldsymbol{\sigma}_{eff}$ during plastic deformation has the same trend similar for $\boldsymbol{\sigma}_y$. Again, when the matrix is passing through the yielding point ($\boldsymbol{\sigma}_Y = 250\,\mathrm{MPa}$), there is an abrupt rebuilt of the stress distribution from the FEM calculation. After completely entering into the plastic range, the change of the distribution curves follows again the classical way, which means the local fields become more inhomogeneous during plastic deformation.

Now we come back to the abnormal change of the stress distribution near the matrix's yielding point revealed by FEM simulation. This can be clarified by zooming several stress distribution curves of close states near the yielding point of the matrix material. Two distributions of $\boldsymbol{\sigma}_{eff}$ in such states are shown in Fig. 7; clearly we observe that when the stress in the matrix reaches the yielding point, another peak develops in the stress range larger than the initial yielding stress (see the first column of Fig. 7). This new peak implies that another group of points in the matrix with similar stress state will develop. With increase of loading, the matrix material passes through the yielding point gradually. The newly developed peak grows up, and the original one fades away (see the second column of Fig. 7). Finally, when all the matrix material comes to yield, the original peak vanishes, and the new one is completely formed, which is shown in Fig. 8.

6 Conclusions

In this paper the local field distribution of a planar composite under plastic deformation was considered. An analytical micromechanical method based on the maximum information theory and secant moduli method has been proposed. This method can predict not only the average stress (strain) in the phase material but also the stress variation. The predicted results with the proposed analytical method compare favorably with finite element simulation. It is shown that with increase of loading the stress distribution curves in the nonlinear matrix become more

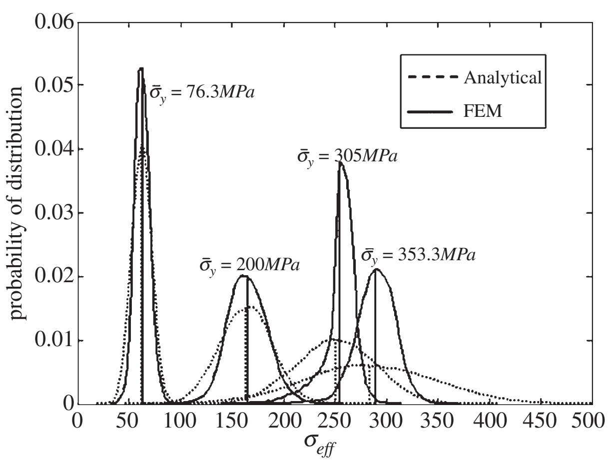

Fig. 6. Probability distribution of von Mises stress at different loading levels

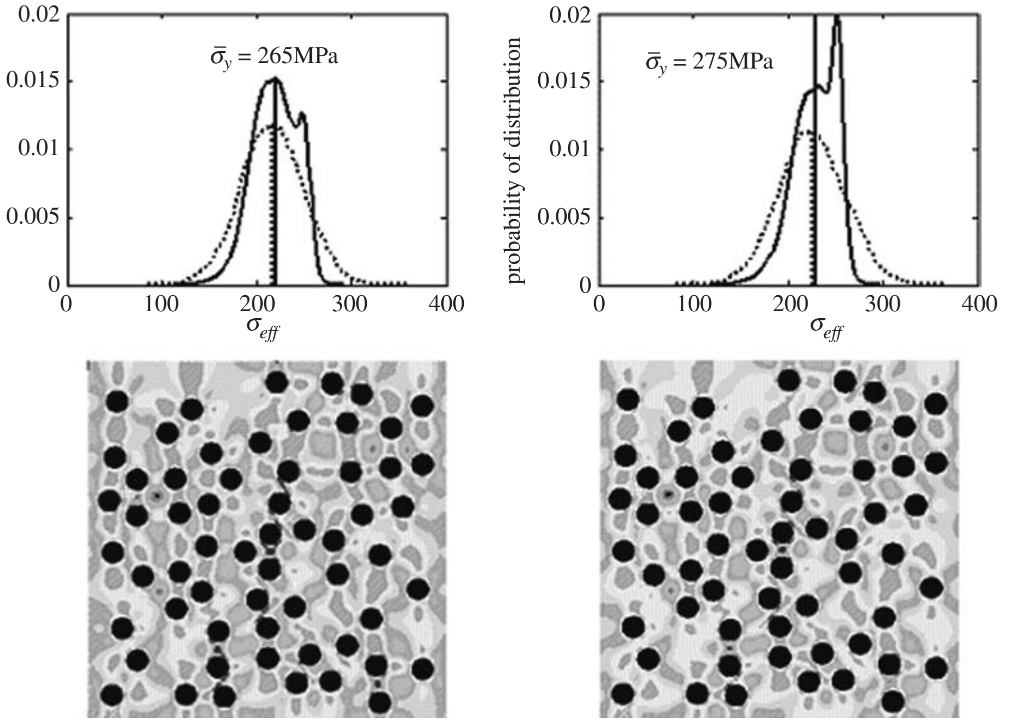

Fig. 7 Distribution of von Mises stress in the matrix during yielding process: a new peak of distribution is developed

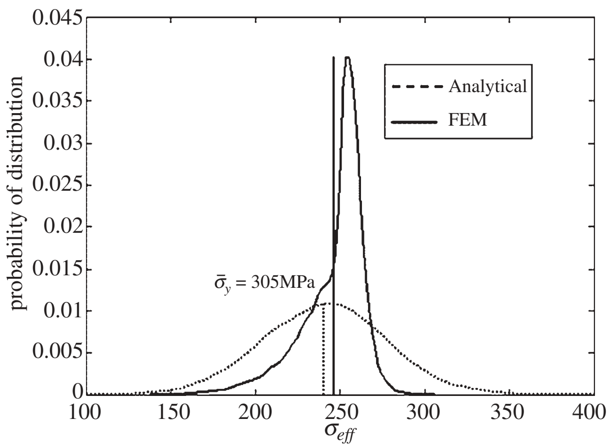

Fig. 8. Distribution of von Mises stress in the matrix: new peak of distribution is fully formed

smooth and flat, indicating that the stress in the matrix becomes more and more inhomogeneous.

Acknowledgements

This work is supported by National Nature Science Foundation of China through Grants 10325210, 10502007

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