Numerical Study on Left-Handed Materials Made of Ferrite and Metallic Wires *

CAI Xiao-Bing(蔡小兵)[1], ZHOU Xiao-Ming(周潇明)[1], HU Geng-Kai(胡更开)[1,2**]

[1]*Department of Applied Mechanics, Beijing Institute of Technology, Beijing 100081*

[2]*National Key Laboratory for the Prevention and Control of Explosion Disaster, Beijing Institute of Technology, Beijing 100081*

(Received 22 May 2005)

Due to coupling effect, we show that it is difficult to realize the left-handed material by placing metallic wires directly into a ferrite matrix. However by introducing an insulating material round the metallic wires to decouple the direct interaction between the metallic wire and ferrite matrix, we have proposed two microstructures, which are shown by numerical simulation to have negative refractive indexes. The influence of microstructure on the transmission property is also examined.

PACS: 41.20.Jb, 78.20.Ci, 76.50.+g

In the past few years, left-handed material (LHM) with negative index of refraction[1] has given rise to intense research activity, due to its potential applications in imaging[2] and many other fields.[3,4] The first idea of the material with both negative permittivity and negative permeability was proposed by Veselago in 1968, however only in 2001 was a left-handed material realized for the first time by using SRR (split-ring resonance) and periodically arranged metallic wires (WIRE).[5] Since then, the research on the LHMs has been intensified, new LHM models have been proposed,[6–8] and the theoretical analyses on the energy transmission and material parameter identification for the LHMs have been performed.[9,10]

In Veselago's pioneering paper,[1] he suggested use of conductive ferrite to provide both negative permittivity and negative permeability for the left-handed material. Many works have followed this idea to fabricate new left-handed materials.[8,11,12] In all of these works, the metal is considered as plasma. However, no experimental or numerical computation was performed to verify these results. The essence of the WIRE arrays to have a negative permittivity is that during plasma resonance, the current induced by moving charges through the WIRE by an electric field exceeds the current induced directly by the electric field on the background, and these two currents flow in opposite direction. However when WIRE is surrounded directly by a material with negative permeability, these two currents are of the same direction, in this case the WIRE arrays have a positive permittivity. Thus, the model of WIRE directly adding into a ferrite matrix is difficult to form an LHM. We will analyse this effect in detail through numerical simulation.

We take Dewar's model (Fig. 1) [13] as an example to show that the WIRE structure is hard to obtain a negative permittivity in a ferrite background. Under a static magnetic field, the ferrite has a tensorial permeability:[11]

$$\begin{bmatrix} \mu & i\kappa & 0 \\ -i\kappa & \mu & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{1}$$

where $\mu = 1 + \dfrac{\gamma^2 H_0 M_s}{\gamma^2 H_0^2 - \omega^2}$, $\kappa = \dfrac{\gamma M_s \omega}{\gamma^2 H_0^2 - \omega^2}$, γ is the gyromagnetic ratio, M_s is the saturation magnetization, H_0 is the applied static magnetic field, ω is the wave frequency. When the wave vector and the magnetic field of the TEM wave are perpendicular to H_0, the effective relative permeability for the ferrite material can be written in the form

$$\mu_r = (\mu^2 - \kappa^2)/\mu. \tag{2}$$

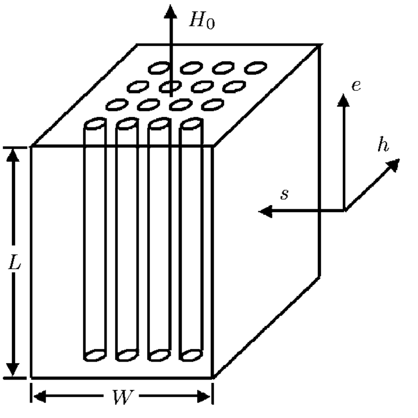

Fig. 1. WIRE structure in ferrite matrix proposed by Dewar.[13] The TEM wave is incident along the s direction, electrical field and static magnetic field are parallel to the WIRE.

Pendry[14] proposed to idealize the WIRE structure as plasma, and he introduced the concept of effective mass of electrons:

$$m_{\rm eff} = \frac{\mu_0 \pi r^2 e^2 n}{2\pi} \ln(a/r), \tag{3}$$

* Supported by the National Natural Science Foundation of China under Grant No 10325210.

** To whom correspondence should be addressed. Email: hugeng@bit.edu.cn

where μ_0 is the matrix permeability, r is the WIRE radius, a is the size of unit cell, e is the electron charge, n is the active electron density in the WIRE. The plasma frequency for the WIRE structure is estimated by

$$\omega_p^2 = n_{\rm eff} e^2/\varepsilon_0 m_{\rm eff}, \tag{4}$$

where $n_{\rm eff} = n\pi r^2/a^2$. Substituting Eq. (4) into the Drude model, the effective dielectric constant of the WIRE structure is provided by

$$\varepsilon_{\rm eff} = 1 - \frac{\omega_p^2}{\omega(\omega + i\varepsilon_0 a^2\omega_p^2/\pi r^2\sigma)}, \tag{5}$$

where σ is the conductivity of the metal.

In Eq. (5), it is found that when $\omega^2 < \omega_p^2$, we have $\mathrm{Re}(\varepsilon_{\rm eff}) < 0$. However, when the WIRE is embedded in a ferrite matrix under a constant applied static magnetic field, in the range of negative permeability of the ferrite, the effective permittivity of the WIRE may not be negative due to the coupling effect. Back to formula (3), due to the negative permeability of the ferrite matrix, the effective mass of electrons has the form

$$m_{\rm eff} = \frac{\mu_r\mu_0\pi r^2 e^2 n}{2\pi}\ln(a/r) < 0, \tag{6}$$

where μ_r is the relative permeability of the ferrite, and $\mu_r < 0$. Substituting Eq. (6) into Eqs. (4) and (5), we observe that $\omega_p^2 < 0$ (the complex frequency has been discussed by Wu *et al.*[17]), so no matter whether $\omega^2 < |\omega_p^2|$ or $\omega^2 > |\omega_p^2|$, we have $\mathrm{Re}(\varepsilon_{\rm eff}) > 0$.

It can be concluded that if the WIRE is embedded into a matrix with a negative permeability, due to the coupling effect, it is difficult to obtain a negative effective permittivity of the composite. Dewar[8] arrived at the same conclusion by solving a boundary value problem.

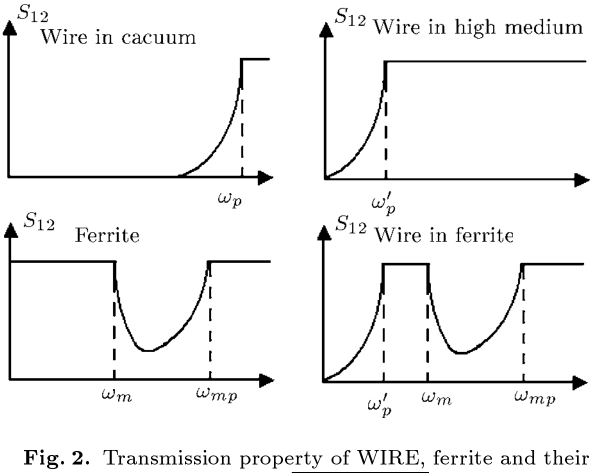

Fig. 2. Transmission property of WIRE, ferrite and their composite. Here $\omega_m = \gamma\sqrt{H_0(H_0+M_s)}$ is the ferromagnetic resonance frequency, and $\omega_{mp} = \gamma(H_0+M_s)$ is the anti-resonance frequency.

In addition, as ferrite usually possesses high relative permittivity, $\varepsilon_r\varepsilon_0$ should be utilized to replace ε_0 in Eq. (4). This significantly reduces the plasma frequency of the WIRE, typically to one tenth of that in vacuum.

Consequently, the frequency range where the ferrite has negative permeability is higher than the range necessary for a negative permittivity of the WIRE. This is illustrated in Fig. 2, due to the coupling effect, usually there is no overlap in the frequency for both negative permittivity and permeability for the composite, necessary to form a left-handed material.

From the above analysis, the key point to make LHM from ferrite and WIRE is to decouple this direct electromagnetic interaction. To this end, the following three modifications can be proposed to reduce this interaction, as shown in Fig. 3. Model A (proposed first by Dewar[8]) is that the WIRE is surrounded by an insulating layer, and the whole cell is together placed into a ferrite matrix. Model B is that the WIRE is placed into an insulating matrix, they are together laid with ferrite vertically, model C is that the WIRE is placed in an insulating matrix, and then they are together laid with ferrite horizontally. In all these three models, a static magnetic field is applied along the Y direction. From the point of fabrication, models B and C are more easy to manipulate than model A. Model C is similar to SRR plus a WIRE structure as proposed by Shelby *et al.*,[5] where the SRR is replaced by the ferrite material. In the following, we analyse the transmission property of these materials.

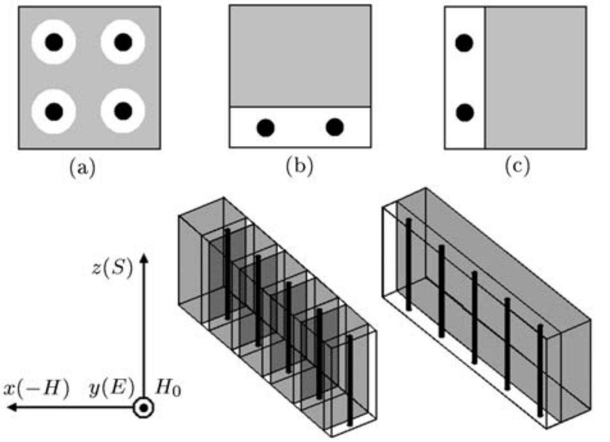

Fig. 3. Top view of models to reduce electromagnetic coupling. The TEM wave is incident along the z direction, with magnetic field parallel to x, the electric field and applied static magnetic field parallel to y.

Finite element calculation on a ferrite material with known property is first performed to check the numerical method. Consider a ferrite of $3p\times6\times15$ mm, which lies in a parallel plate waveguide (Fig. 4), where p is the number of the element repeated in the x direction. The waveguide is a vacuum and two sides of the x direction are set to be master–slave boundary condition (periodic condition) with zero phase delay. Of all models in this study, the ferrite is supposed to have a saturation magnetization of 1700 G, and a relative dielectric constant 13. The applied static magnetic field is 1256 Oe.

When a TEM wave is incident along the z direction, with magnetic field parallel to the x direction, and electric field parallel to the y direction, substituting ferrite's tensorial permeability (Eq. (1)) into plane wave propagation equation, and noting $k = k_x$, then S-parameters can be estimated analytically, and they are given by the following form:[10]

$$S_{11} = \frac{R(1-T^2)}{1-R^2T^2},$$
$$S_{21} = \frac{T(1-R^2)}{1-R^2T^2}, \tag{7}$$

where $R = (z-1)/(z+1)$; $T = \exp(-i\sqrt{\varepsilon_r\mu_r}k_0 d)$; μ_r is the ferrite effective permeability given by Eq. (2); k_0 is the vacuum wave vector; d is the thickness of model. Figure 5 gives the S-parameter S_{21} estimated from Eq. (7) and calculated by the HFSS method.[15]

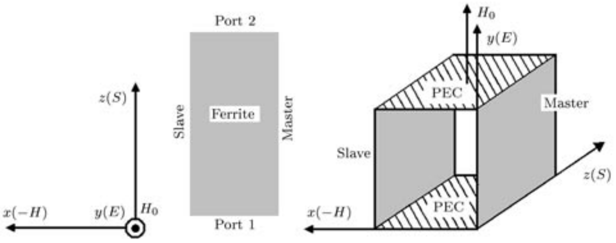

Fig. 4. Finite element model for a plate waveguide, periodic condition along the x direction. Top and bottom surfaces in the y direction are PEC, front and back surfaces in the z direction are two waveports.

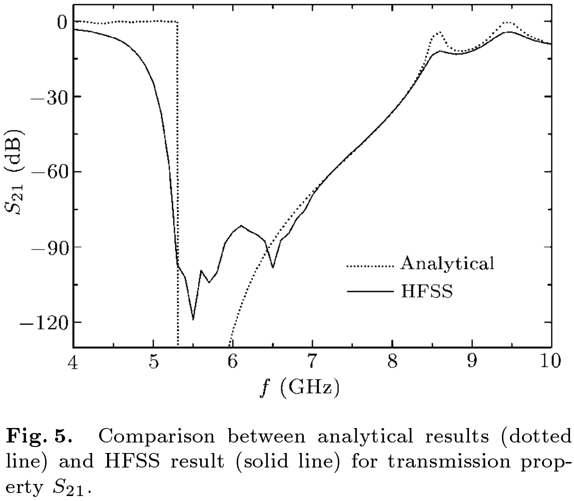

Fig. 5. Comparison between analytical results (dotted line) and HFSS result (solid line) for transmission property S_{21}.

As shown in Fig. 5, both the analytical and HFSS results for the S_{21} parameter predict stopband extending from 5.3 to 8.2 GHz, in which S_{21} is less than −30 dB (see Fig. 5). The results predicted by the two methods agree well in a large part of frequency range. The curve rises less shapely at the right part within the stopband, this is due to a small negative permeability of the ferrite at these frequencies. The applied static magnetic field also interferes with the stopband, which is explained in Fig. 2.

The finite element model for the new proposed structures proposed previously is the same as shown in Fig. 4. The transmission property S_{21} is calculated for the three microstructures, the computed results are shown in Fig. 6, the result of Dewar's initial model (Fig. 1) is also included for comparison. Compared to the pure ferrite material (Fig. 5) and Dewar's initial model, it is found that there exist some passbands for the composites (models A, B, C) approximately spanning from 6.75 to 7.75 GHz, originally these passbands lie within the stopbands of the ferrite matrix, however the initial model (Fig. 1) proposed by Dewar has no passband. This indicates that the proposed composites have both negative permittivity and permeability. We have checked from the phase diagram, indeed in this frequency range, the composites are left-handed materials. In the three models, the ferrite occupies a volume function of 66%, 73% and 67%, respectively. The radii of all metallic wires are 0.15 mm.

The simulated results show that by including an insulating material round metallic wires, the electromagnetic coupling effect is greatly reduced. This makes left-handed materials possible from ferrite and WIRE. According to Eq. (4), the presence of large volume fraction of ferrite lowers the plasma frequency of the WIRE typically from 50 to 7 GHz, this is why S_{21} is still very high for the frequency above 8.2 GHz. When the frequency is slightly larger than the resonance frequency of the ferrite (5.3 GHz), the absolute value of the ferrite permeability is reduced quickly, the effective permeability of the element is dominated by the WIRE and insulating material.

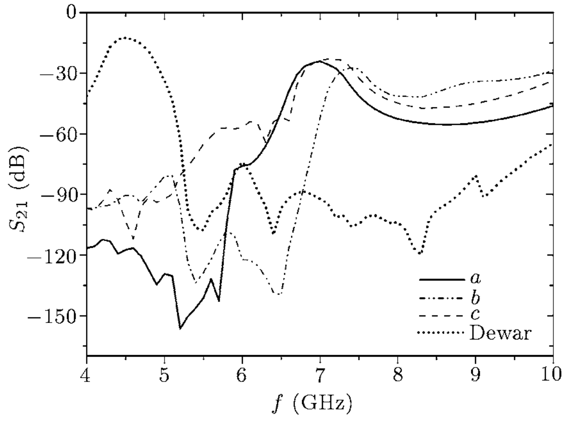

Fig. 6. Simulated transmission property S_{21} for models A, B, and C, and Dewar's initial model.

Figure 7 shows the electric field variation as a function of time along the central line (along the z direction) of the models A, B and C, respectively. For the electric field $E_0 \exp(i(kz-wt))$, let the phase be a constant (here we take to be 0), we can obtain constant phase diagram, which can be written as[16]

$$z = \frac{1}{k}\omega t. \tag{8}$$

The slopes of the black line in Fig. 7 are negative, indicating that their wave vector k is negative. Refractive indexes of the composites can be estimated directly from Fig. 7 by k, these results are listed in Table 1, the complex refractive indexes can also be evaluated directly from transmission property S_{11} and S_{21},[10] they are also listed for comparison. A good agreement is observed.

In order to have an optimal design for the microstructure, the influence of microstructure parameters on the transmission property is examined, the computed S_{21} as a function of the ferrite volume fraction and the radius of WIRE for model C at 7.0 GHz is shown in Fig. 8. In the computation, the WIRE is assumed to be placed in the middle of insulating material. When the radius of the WIRE varies from 0.10 to 0.24 mm, S_{21} is only slightly reduced. It is also found that S_{21} reaches a maximum value at 60% volume fraction of ferrite (the 1.8 mm width of the ferrite in the cell). Thus optimal design of the microstructure for the left-handed material will be useful, and this will be a subject of our future works.

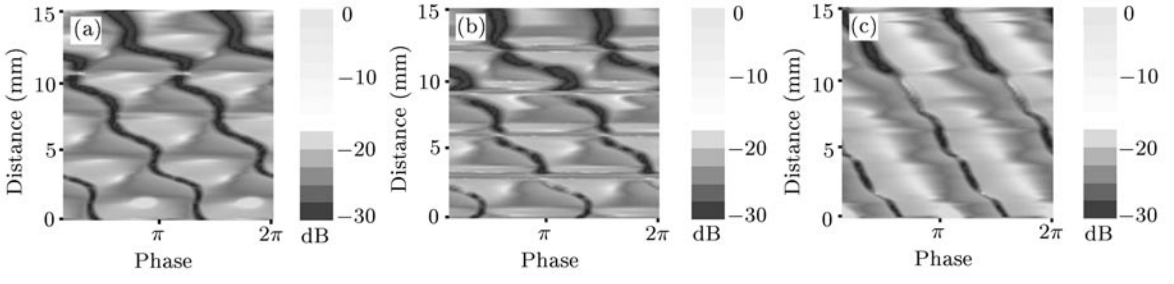

Fig. 7. Electric field variation through the centre of models A, B, and C, at frequencies of 7.0, 7.2 and 7.0 GHz, respectively.

Table 1. Refractive indexes $n'_{\rm eff}$ estimated from the S parameters and n' obtained from Fig. 7.

Model	f (GHz)	S'_{11}	S''_{11}	S'_{21}	S''_{21}	$n'_{\rm eff}$	$n''_{\rm eff}$	n'
A	7.00	−0.84	0.09	0.05	−0.003	−3.09	0.87	−3.0
B	7.20	−0.86	0.23	0.03	0.0006	−3.10	0.87	−3.0
C	7.00	−0.60	−0.04	0.07	−0.01	−2.88	0.99	−2.9

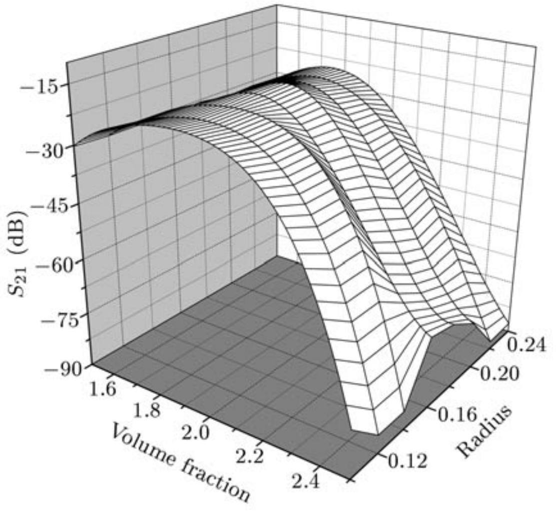

Fig. 8. Influence of ferrite volume fraction and WIRE radius on transmission property S_{21}.

In conclusion, we have shown that due to the electromagnetic coupling effect, it is difficult to make left-handed materials directly by embedding metallic wires into ferrite matrix. By introducing an insulating material round the WIRE, we have proposed two new microstructures made of WIRE, insulating material and ferrite, which are shown by numerical simulation to have negative refractive indices in certain frequency.

References

[1] Veselago V G 1968 *Sov. Phys. Usp.* **10** 509
[2] Pendry J B 2000 *Phys. Rev. Lett.* **85** 3966
[3] Schwartz B T and Piestun R 2004 *Appl. Phys. Lett.* **85** 1
[4] Li J, Zhou L, Chan C T and Sheng P 2003 *Phys. Rev. Lett.* **90** 8
[5] Shelby R A, Smith D R and Schultz S 2001 *Science* **292** 77
[6] Sanada A, Caloz C and Tatsuo I 2004 *IEEE T. Microwave Theory* **52** 1252
[7] Chen H S, Ran L X, Huangfu J T, Zhang X M, Chen K S, Grzegorezyk T M and Kong J A 2004 *Phys. Rev.* E **70** 057605
[8] Dewar G 2003 *Proc. SPIE* **5218** 140
[9] Cui T J and Kong J A 2004 *Phys. Rev.* B **70** 205106
[10] Chen X D, Grzegorezyk T M, Wu B L, Pacheco J and Kong J A 2004 *Phys. Rev.* E **70** 016608
[11] Chui S T and Hu L B 2000 *Phys. Rev.* B **65** 144407
[12] Wu R X 2005 *J. Appl. Phys.* **97** 076105
[13] Dewar G 2000 *Int. J. Mod. Phys.* B **15** 3258
[14] Pendry J B 1996 *Phys. Rev. Lett.* **76** 4773
[15] Ansoft HFSS (High Frequency Structure Simulator) http://www.ansoft.com.cn
[16] Moss C D, Grzegorezyk T M, Zhang Y and Kong J A 2002 *Prog. Electromagn. Res.* **35** 315
[17] Wu L, He S and Shen L 2003 *Phys. Rev.* B **67** 235103