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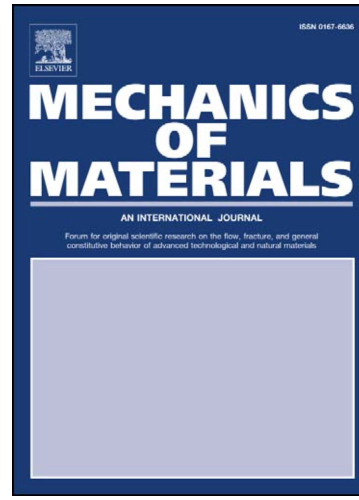

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Overall plasticity of micropolar composites with interface effect

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Abstract:

Overall property of composite materials depends on particle size while its volume fraction is kept constant. A micromechanical method is proposed to predict the size-dependent plastic property for composite materials, the proposed method takes into account the nonlocal effect by idealizing the matrix as a micropolar material, and the interface effect between different phases is also considered. A perturbation method for a micropolar composite with the interface effect is established by a rigorous energy equivalent method, it is then used to estimate the average second order stress/couple stress moment in the local phase. A secant modulus scheme is proposed to predict the overall nonlinear behavior for a micropolar composite with the interface effect. It is found that the nonlocal and the interface effects on the size-dependent yielding and strain hardening behavior may be synchronized or desynchronized depending on the nature of the interface. For a hydrostatic loading, it is found that the interface effect has an important influence on the overall yielding of the composite. The instability of the composite induced by interface effect is also discussed.

Keywords: Micromechanics; Micropolar theory; Interface effect; Plasticity

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1. Introduction

Size-dependent overall property for composite materials has been well observed by experiment when the size of reinforced phase varies and the other microstructure parameters are kept unchanged (Kouzeli and Mortensen, 2002). Another size-dependent phenomenon recently analyzed comes from interface effect, when the dimension of a structure is reduced, the surface to volume ratio becomes important (Miller and Shenoy, 2000).

Different techniques to establish the relation between the effective property and the microstructure for a heterogeneous material have been developed, which are summarized in references (Nemat-Nasser and Hori, 1993, Milton, 2002, Hashin, 1983, Buryachenko, 2001, Hu and Weng, 2000). However, due to the absence of any explicit length scale in the basic equations, the classical homogenization approach fails to predict the size-dependent effective property. To circumvent this difficulty, many models have been developed. Two different analytical approaches based on continuum formulation are proposed: for the first approach, the constituent materials are idealized as high order continuum, due to the fact that the separation of length scales is impossible and the nonlocal effect becomes important; the other approach argues that the interface effect comes into play, when the surface-to-volume ratio is not negligible. For the first approach, the strain gradient (Smyshlyaev and Fleck, 1994) or micropolar (Liu and Hu, 2005, Hu *et al.*, 2005) models have been incorporated into proper micromechanical models, the size-dependent overall elastic and plastic properties for composite materials can be predicted. An intrinsic length is introduced which is of micrometer scale, this length scale is believed to be related to the microstructure of the constituent materials (Hu *et al.*, 2005). In the second approach, the constituent materials are assumed to be local in nature, however the stress discontinuity is allowed across the interface between the matrix and the reinforced phase, and this discontinuity is governed by Young-Laplace equations (Sharma and Ganti, 2004, Duan *et al.*, 2005). It is emphasized that here the interface model refers more precisely to the *interface stress* model or *interface elasticity* (Ibach, 1997, Gurtin and Murdoch, 1975), which is well accepted as one of the origins for size-dependent behavior. Other interface models with displacement or strain jumps, which are widely employed to model damage, debonding or strain localization, are not considered in this paper. The effective modulus predicted by the interface model depends also on particle size, an intrinsic length scale is also introduced. It seems that this size effect is only pronounced when the particles are within nanometer scale (Sharma *et al.*, 2003). Clearly, the nonlocal and interface effects have sound and different physical backgrounds. When the size of reinforced phase becomes small, both the interface and the nonlocal effects may become important. So it is interesting to propose a micromechanical model which can simultaneously consider these two effects. Recently Chen *et al.*(2007) propose an elastic micromechanical model in framework of micropolar theory with the interface effect. However, the nonlinear behavior of a micropolar composite with the interface effect has not been addressed.

In this paper, an analytical approach is proposed to estimate elastoplastic properties for a micropolar composite with the interface effect. The manuscript is arranged as follows: the estimation of the overall elastic moduli for a micropolar composite with the interface effect will be briefly outlined in section II; the perturbation relation for a heterogeneous micropolar material with interface stress jump will be developed and the second order stress(couple) moment of the matrix phase will be derived in section III; In section IV, a secant modulus scheme will be proposed to determine the overall nonlinear response for the composite material, numerical examples will also be presented; the paper is closed by some conclusions.

2. Overall elastic modulus for micropolar composite with interface effect

Recently, Chen *et al.* (2007) proposed an analytical method to estimate the overall elastic moduli of micropolar composites including interface effect, the basic concept will be summarized in the following.

The geometrical, balance and constitutive equations for a centro-symmetric and isotropic micropolar continuum in the absence of body force and moment are given by Eringen(1999) and Nowacki(1986):

$$\boldsymbol{\varepsilon} = \nabla \otimes \mathbf{u} - \mathbf{e} \cdot \boldsymbol{\varphi}, \quad \mathbf{k} = \nabla \otimes \boldsymbol{\varphi}, \tag{1a}$$

$$\nabla \cdot \boldsymbol{\sigma} = 0, \quad \nabla \cdot \mathbf{m} + \mathbf{e} : \boldsymbol{\sigma} = 0, \tag{1b}$$

$$\boldsymbol{\sigma} = \lambda \mathrm{Tr}(\boldsymbol{\varepsilon})\mathbf{I} + (\mu + \kappa)\boldsymbol{\varepsilon} + (\mu - \kappa)\boldsymbol{\varepsilon}^T, \quad \mathbf{m} = \alpha \mathrm{Tr}(\mathbf{k})\mathbf{I} + (\beta + \gamma)\mathbf{k} + (\beta - \gamma)\mathbf{k}^T \tag{1c}$$

where $\boldsymbol{\sigma}$ and $\mathbf{m}$ are respectively non-symmetric stress and couple stress tensors, $\boldsymbol{\varepsilon}$ and $\mathbf{k}$ are strain and torsion tensors, $\mathbf{u}$ and $\boldsymbol{\varphi}$ are displacement and micro-rotation vectors, $\mathbf{e}$ is the permutation tensor, $\mu, \lambda, \kappa, \gamma, \beta, \alpha$ are the six independent elastic constants for an isotropic micropolar material， $\mathbf{I}$ represents the 2nd rank unit tensor in a three-dimensional space，the superscript T means the transposition of a tensor. A well-posed problem is closed by the following boundary conditions:

$$\mathbf{N} \cdot \boldsymbol{\sigma} = \bar{\mathbf{t}}, \mathbf{N} \cdot \mathbf{m} = \bar{\mathbf{p}} \ \text{ on } \partial V_\sigma, \quad \mathbf{u} = \bar{\mathbf{u}}, \boldsymbol{\varphi} = \bar{\boldsymbol{\varphi}} \ \text{ on } \partial V_u. \tag{2}$$

where $\bar{\mathbf{t}}$ and $\bar{\mathbf{p}}$ are the prescribed force and couple on the boundary ∂V_σ, $\bar{\mathbf{u}}$ and $\bar{\boldsymbol{\varphi}}$ are separately the prescribed displacement and micro-rotation on the boundary ∂V_u, $\mathbf{N}$ is outward unit normal on the boundary.

For a micropolar composite with the interface effect, jumps for the stress and couple, denoted by $[\boldsymbol{\sigma}]$ and $[\mathbf{m}]$, respectively, are allowed across the interface between the matrix and the particle. The interface constitutive equation and Yang-Laplace equation for a micropolar media can be written as (Chen *et al.*, 2007):

$$\boldsymbol{\sigma}_s = \lambda_s \mathrm{Tr}(\boldsymbol{\varepsilon}_s)\mathbf{I}^{(2)} + (\mu_s + \kappa_s)\boldsymbol{\varepsilon}_s + (\mu_s - \kappa_s)\boldsymbol{\varepsilon}_s{}^T, \tag{3a}$$

$$\mathbf{m}_s = \alpha_s \mathrm{Tr}(\mathbf{k}_s)\mathbf{I}^{(2)} + (\beta_s + \gamma_s)\mathbf{k}_s + (\beta_s - \gamma_s)\mathbf{k}_s{}^T, \tag{3b}$$

$$\mathbf{n}\cdot[\boldsymbol{\sigma}]\cdot\mathbf{P} = -\nabla_s\cdot\boldsymbol{\sigma}_s, \quad \mathbf{n}\cdot[\boldsymbol{\sigma}]\cdot\mathbf{n} = -\boldsymbol{\sigma}_s : \mathbf{b} \tag{4a}$$

$$\mathbf{n}\cdot[\mathbf{m}]\cdot\mathbf{P} = -\nabla_s\cdot\mathbf{m}_s, \quad \mathbf{n}\cdot[\mathbf{m}]\cdot\mathbf{n} = -(\mathbf{m}_s : \mathbf{b} + \boldsymbol{\sigma}_s : \mathbf{e}_s) \tag{4b}$$

where $\boldsymbol{\varepsilon}_s = \mathbf{P}\cdot\boldsymbol{\varepsilon}\cdot\mathbf{P}$, $\mathbf{k}_s = \mathbf{P}\cdot\mathbf{k}\cdot\mathbf{P}$ denote the projections of the strain and torsion tensors onto the tangent plane of the interface, namely, the interface strain and interface torsion. The projection tensor is defined by $\mathbf{P} = \mathbf{I} - \mathbf{n}\otimes\mathbf{n}$. Similarly $\boldsymbol{\sigma}_s$ and $\mathbf{m}_s$ are the interface stress and couple stress, $\mathbf{n}$ is unit normal of the interface, $\mathbf{I}^{(2)}$ represents the 2nd rank unit tensor in two-dimensional space, ∇_s and $\mathbf{e}_s$ are the gradient operator and permutation tensor on the interface, $\mathbf{b}$ is the curvature tensor of the interface, ($\lambda_s\ \beta_s\ \gamma_s\ \alpha_s\ \mu_s\ \kappa_s$) are the six micropolar interface material constants.

For a two-phase micropolar composite, if we consider only the classical effective property (the symmetric part of the effective modulus), then the micromechanical procedure can be greatly simplified (Liu and Hu, 2005). To this end, the following tractions will be applied on the boundary of a representative volume element (RVE):

$$\mathbf{N}\cdot\boldsymbol{\sigma} = \mathbf{N}\cdot\boldsymbol{\Sigma}^{sym}, \quad \mathbf{N}\cdot\mathbf{m} = 0. \tag{5}$$

Then the classical effective stiffness and compliance tensor $\mathbf{C}_C^{sym}$, $\mathbf{M}_C^{sym}$ can be defined as:

$$<\boldsymbol{\sigma}^{sym}> = \mathbf{C}_C^{sym} : <\boldsymbol{\varepsilon}^{sym}>, \qquad <\boldsymbol{\varepsilon}^{sym}> = \mathbf{M}_C^{sym} : <\boldsymbol{\sigma}^{sym}>. \tag{6}$$

where $<\bullet>$ means the volume average on the RVE, the subscript *sym* means the symmetric part of the corresponding quantity. Such scheme represents a homogenization from high-order continua to a Cauchy one, and it remains valid under the following size relations: the particle size is comparable to the intrinsic length of the matrix, and there is clear size separation between the RVE and the macroscopic structure. For such a special case, the Hill's condition holds, *i.e.* both the static and kinematics boundary conditions exist and they give an equivalent definition for the overall Cauchy material modulus. Since the displacement is continuous across the interface and the stress has a jump, then the volume average of the local stress and the strain over the RVE reads:

$$<\boldsymbol{\sigma}^{sym}>=(1-f)<\boldsymbol{\sigma}^{sym}>_0+f<\boldsymbol{\sigma}^{sym}>_1+\frac{1}{2V}\int_\Gamma\{(\mathbf{n}\cdot[\boldsymbol{\sigma}^{sym}])\otimes\mathbf{x}+\mathbf{x}\otimes(\mathbf{n}\cdot[\boldsymbol{\sigma}^{sym}])\}d\Gamma$$

$$=\frac{1}{2V}\{\int_S(\mathbf{N}\cdot\boldsymbol{\Sigma}^{sym})\otimes\mathbf{x}dS+\int_S\mathbf{x}\otimes(\mathbf{N}\cdot\boldsymbol{\Sigma}^{sym})dS\}=\boldsymbol{\Sigma}^{sym} \quad (7a)$$

$$<\boldsymbol{\varepsilon}^{sym}>=(1-f)<\boldsymbol{\varepsilon}^{sym}>_0+f<\boldsymbol{\varepsilon}^{sym}>_1 \quad (7b)$$

Where V is the volume of the RVE, Γ denotes the interface between the inclusion and the matrix, S represents the external surface of the RVE, f is volume fraction of the inclusion. $<\bullet>_i$ means the volume average over the ith ($i=0,1$) phase. For the previous traction boundary condition $\boldsymbol{\Sigma}^{sym}$, $<\sigma^{sym}>_i$ is determined by a concentration factor $\mathbf{P}_i^{sym}$ as

$$<\boldsymbol{\sigma}^{sym}>_i=\mathbf{P}_i^{sym}:\boldsymbol{\Sigma}^{sym} \quad (8)$$

Further more, if we define P_s^{sym} such that:

$$\frac{1}{2V_1}\int_\Gamma\{(\mathbf{n}\cdot[\boldsymbol{\sigma}^{sym}])\otimes\mathbf{x}+\mathbf{x}\otimes(\mathbf{n}\cdot[\boldsymbol{\sigma}^{sym}])\}d\Gamma=\mathbf{P}_s^{sym}:\boldsymbol{\Sigma}^{sym}, \quad (9)$$

Once $\mathbf{P}_i^{sym}$ and $\mathbf{P}_s^{sym}$ are obtained, the classical overall compliance tensor of the micropolar composite can be evaluated by equations(6-9), leading to:

$$\mathbf{M}_C^{sym}=\mathbf{M}_0^{sym}+f(\mathbf{M}_1^{sym}-\mathbf{M}_0^{sym}):\mathbf{P}_1^{sym}-f\mathbf{M}_0^{sym}:\mathbf{P}_s^{sym}, \quad (10)$$

where $\mathbf{M}_0^{sym}$ and $\mathbf{M}_1^{sym}$ are the symmetric part of the compliance tensors for the matrix and the inclusion respectively. In this paper, Mori-Tanaka's method is used to estimate the concentration factors $\mathbf{P}_i^{sym}$ and $\mathbf{P}_s^{sym}$, in turn, the effective compliance tensor. Its analytical expressions for a cylindrical fiber reinforced composite are given in Appendix, see also Chen *et al*. (2007) for more detail.

3. Local effective stress by perturbation method

In the following, the fiber is assumed to be elastic. To determine the onset and evolution of plasticity for a micropolar composite, the average effective stress of the matrix material must be determined and related to the applied macroscopic load $\mathbf{\Sigma}^{sym}$. This can be fulfilled by a perturbation method (Kreher and Pompe, 1989, Qiu and Weng, 1992, Hu, 1996 for classical composites; Liu and Hu, 2005 for a micropolar composite). Consider a two-phase micropolar composite material with the interface effect, the traction boundary conditions defined by equation (5) are applied on the boundary of the RVE. Due to the contribution of the interface, the energy density averaged over the RVE can be written as:

$$w = <\boldsymbol{\sigma} : \boldsymbol{\varepsilon} + \mathbf{m} : \mathbf{k}> + \frac{1}{V}\int_{\Gamma}(\boldsymbol{\sigma}_s : \boldsymbol{\varepsilon}_s + \mathbf{m}_s : \mathbf{k}_s)d\Gamma . \qquad (11)$$

With help of equilibrium equations (1) and generalized Yang-Laplace equations (4), equation (11) can be further written as:

$$w = \frac{1}{V}\int_{V_0}[\nabla\cdot(\boldsymbol{\sigma}\cdot\mathbf{u}) + \nabla\cdot(\mathbf{m}\cdot\boldsymbol{\varphi})]dV + \frac{1}{V}\int_{V_1}[\nabla\cdot(\boldsymbol{\sigma}\cdot\mathbf{u}) + \nabla\cdot(\mathbf{m}\cdot\boldsymbol{\varphi})]dV$$
$$+\frac{1}{V}\int_{\Gamma}[\mathbf{n}\cdot[\boldsymbol{\sigma}]\cdot\mathbf{u} + \mathbf{n}\cdot[\mathbf{m}]\cdot\boldsymbol{\varphi}]d\Gamma + \frac{1}{V}\int_{\Gamma}[\nabla_s\cdot(\boldsymbol{\sigma}_s\cdot\mathbf{u}) + \nabla_s\cdot(\mathbf{m}_s\cdot\boldsymbol{\varphi})]d\Gamma ,$$

where V_0 and V_1 are the volumes of the two phase, respectively. By using the divergence theorem on the first two terms of the right hand side of the above equation and the surface divergence theorem on the last item, we have:

$$w = \frac{1}{V}\int_{S}\{\mathbf{N}\cdot(\boldsymbol{\sigma}\cdot\mathbf{u}) + \mathbf{N}\cdot(\mathbf{m}\cdot\boldsymbol{\varphi})\}dS + \frac{1}{V}\int_{\Gamma}\{\mathbf{n}'\cdot(\boldsymbol{\sigma}^{+}\cdot\mathbf{u} + \mathbf{m}^{+}\cdot\boldsymbol{\varphi})\}d\Gamma$$
$$+\frac{1}{V}\int_{\Gamma}\{\mathbf{n}\cdot(\boldsymbol{\sigma}^{-}\cdot\mathbf{u} + \mathbf{m}^{-}\cdot\boldsymbol{\varphi})\}d\Gamma + \frac{1}{V}\int_{\Gamma}\{\mathbf{n}\cdot[\boldsymbol{\sigma}]\cdot\mathbf{u} + \mathbf{n}\cdot[\mathbf{m}]\cdot\boldsymbol{\varphi}\}d\Gamma$$

$$+\frac{1}{V}\int_{\partial\Gamma}\{\tilde{\mathbf{n}}\cdot(\boldsymbol{\sigma}_s\cdot\mathbf{u})+\tilde{\mathbf{n}}\cdot(\mathbf{m}_s\cdot\boldsymbol{\varphi})\}dl$$

where $\mathbf{n}'$ and $\mathbf{n}$ are respectively the outward and inward unit normal of the interface; $\boldsymbol{\sigma}^+$ and $\boldsymbol{\sigma}^-$ are the stresses on each side of the interface; the last item of the right hand side of the above equation is the integration along the interface; $\tilde{\mathbf{n}}$ lies in the tangent plane of the interface and orthogonal to the interface. It is obvious that all except the first item vanish in the above expression. Finally the micro-macro transition of elastic energy for the micropolar RVE with the interface effect can be written as:

$$<\boldsymbol{\sigma}:\boldsymbol{\varepsilon}+\mathbf{m}:\mathbf{k}>+\frac{1}{V}\int_{\Gamma}(\boldsymbol{\sigma}_s:\boldsymbol{\varepsilon}_s+\mathbf{m}_s:\mathbf{k}_s)d\Gamma=\boldsymbol{\Sigma}^{sym}:<\nabla\otimes\mathbf{u}>, \quad (12)$$

Equation(12) holds for any balanced stress field corresponding to (5) and compatible strain field. With help of the phase constitutive equations, equation (12) can be further written as:

$$\boldsymbol{\Sigma}^{sym}:\mathbf{M}_C^{sym}:\boldsymbol{\Sigma}^{sym}=<\boldsymbol{\sigma}:\mathbf{M}:\boldsymbol{\sigma}+\mathbf{m}:\mathbf{L}:\mathbf{m}>+\frac{1}{V}\int_{\Gamma}(\boldsymbol{\sigma}_s:\mathbf{M}_s:\boldsymbol{\sigma}_s+\mathbf{m}_s:\mathbf{L}_s:\mathbf{m}_s)d\Gamma, \quad (13)$$

where $\mathbf{M}$, $\mathbf{L}$, $\mathbf{M}_s$ and $\mathbf{L}_s$ denote the local micropolar compliances and the interface compliances, respectively. In the following, let the macroscopic applied stress fixed and the local compliance tensors have independent variations($\delta\mathbf{M}$, $\delta\mathbf{L}$)，this will lead to the variations of the local stress ($\delta\boldsymbol{\sigma}$ $\delta\mathbf{m}$ $\delta\boldsymbol{\sigma}_s$ $\delta\mathbf{m}_s$), as well as the variation of the effective compliance $\delta\mathbf{M}_C^{sym}$. From equation (13), we have

$$\boldsymbol{\Sigma}^{sym}:\delta\mathbf{M}_C^{sym}:\boldsymbol{\Sigma}^{sym}=<\boldsymbol{\sigma}:\delta\mathbf{M}:\boldsymbol{\sigma}+\mathbf{m}:\delta\mathbf{L}:\mathbf{m}>+2<\boldsymbol{\sigma}:\mathbf{M}:\delta\boldsymbol{\sigma}+\mathbf{m}:\mathbf{L}:\delta\mathbf{m}>$$

$$+\frac{2f}{V_1}\int_{\Gamma}(\boldsymbol{\sigma}_s:\mathbf{M}_s:\delta\boldsymbol{\sigma}_s+\mathbf{m}_s:\mathbf{L}_s:\delta\mathbf{m}_s)d\Gamma$$

With help of equation (12), and the ($\delta\boldsymbol{\sigma}$ $\delta\mathbf{m}$ $\delta\boldsymbol{\sigma}_s$ $\delta\mathbf{m}_s$) are statically equilibrium fields with zero applied load condition, it can be shown that:

$$< \boldsymbol{\sigma} : \mathbf{M} : \delta\boldsymbol{\sigma} + \mathbf{m} : \mathbf{L} : \delta\mathbf{m} >= -\frac{1}{V}\int_{\Gamma}(\boldsymbol{\sigma}_s : \mathbf{M}_s : \delta\boldsymbol{\sigma}_s + \mathbf{m}_s : \mathbf{L}_s : \delta\mathbf{m}_s)d\Gamma$$

Finally, the perturbation relation of a micropolar composite with the interface effect can be obtained as:

$$\boldsymbol{\Sigma}^{sym} : \delta\mathbf{M}_C^{sym} : \boldsymbol{\Sigma}^{sym} =< \boldsymbol{\sigma} : \delta\mathbf{M} : \boldsymbol{\sigma} + \mathbf{m} : \delta\mathbf{L} : \mathbf{m} >. \tag{14}$$

It is found that the perturbation relation is formally identical to that without the interface effect (Liu and Hu, 2005), however, the effective moduli are different. If the micropolar effect is neglected, the classical result with the interface effect can be obtained as a special case, as discussed by Zhang and Wang (2007).

In the following, the inclusion is purely elastic, only the matrix can undergo plastic deformation. Therefore it is assumed that the composite yields when the average effective stress of the matrix reaches its elastic limit σ_y, that is: $<\sigma_{eff}>_0 = \sigma_y$. For a two-dimensional micropolar material, the generalized Mises effective stress can be defined as (Xun *et al*. 2004):

$$\sigma_{eff}^2 = \frac{3}{2}\left(\sigma'_{(\alpha\beta)}\sigma'_{(\alpha\beta)}) + \frac{1}{l_m^2}m_{\alpha 3}m_{\alpha 3}\right) \tag{15}$$

where $(\alpha, \beta = 1,2)$, and $\sigma'_{(\alpha\beta)}$ means the symmetric part of the deviatoric stress, l_m is the intrinsic characteristic length introduced by the micropolar effect, another length scale l_s associated with the interface effect also appears in the overall elastic moduli (see Appendix). Without loss of nonlocal feature, the micropolar plasticity in this paper is simplified to highlight the interaction of the two size-dependent mechanisms, the skew-symmetric part of stress is assumed to not trigger plasticity, and the elastic and plastic micropolar intrinsic lengths are set to be equal. A more general plastic method for a micropolar composite is discussed in Liu and Hu (2005). From the general equation (14), we can let the matrix's moduli μ_0 and $(\beta_0 + \gamma_0)$ have independent variations, which lead to:

$$<\sigma'_{(\alpha\beta)}\sigma'_{(\alpha\beta)}>_0=\frac{1}{1-f}\left[\frac{\mu_0^2}{\mu_C^2}\frac{\partial\mu_C}{\partial\mu_0}\Sigma'_{(\alpha\beta)}\Sigma'_{(\alpha\beta)}+2\frac{\mu_0^2}{k_C^2}\frac{\partial k_C}{\partial\mu_0}\Sigma_m^2\right],$$

$$<m_{\alpha 3}m_{\alpha 3}>_0=\frac{(\beta_0+\gamma_0)^2}{2(1-f)\mu_C^2}\frac{\partial\mu_C}{\partial(\beta_0+\gamma_0)}\Sigma'_{(\alpha\beta)}\Sigma'_{(\alpha\beta)}. \quad (16)$$

Where μ_C , k_C are the effective in-plane shear and bulk moduli for a fiber micropolar composite with the interface effect, their expressions are given in Appendix.

Finally the average Mises stress of the micropolar matrix defined by equation (15) can be evaluated by

$$<\sigma_{eff}^2>_0=\frac{1}{(1-f)}\left[\frac{\mu_0^2}{\mu_C^2}\frac{\partial\mu_C}{\partial\mu_0}+\frac{1}{2}\left(\frac{\beta_0+\gamma_0}{l_{\mathrm{m}}\mu_C}\right)^2\frac{\partial\mu_C}{\partial(\beta_0+\gamma_0)}\right]\Sigma_e^2+\frac{3}{(1-f)}\frac{\mu_0^2}{k_C^2}\frac{\partial k_C}{\partial\mu_0}\Sigma_m^2, \quad (17)$$

where $\Sigma_e^2=3\Sigma'_{(\alpha\beta)}\Sigma'_{(\alpha\beta)}/2$, $\Sigma_m=\Sigma_{\alpha\alpha}/2$ are respectively the overall Mises and hydrostatic stress. Equations (14) or (17) are rigorous and universal to any microstructure, providing that a reasonable estimation of the elastic effective compliance $\mathbf{M}_C$ is available (Mori-Tanaka estimation is adopted in this paper).

4. Nonlinear stress-strain relation for micropolar composite with interface effect

4.1 Secant modulus method with second stress(couple)moment

Plastic deformation will be developed in the matrix when the applied macroscopic stress exceeds the initial yield stress. In order to consider the weakened constraint power of the plastic matrix on the fiber, the secant moduli method based on second order stress and couple stress moment will be utilized (Liu and Hu, 2005). Of cause other linearized approaches can also be utilized (see Dormieux *et al.* 2002), however in this paper we will focus on the interface effect and nonlocal effect.

For the micropolar matrix, a power-law deformation version of plasticity can be written as, with help of the generalized effective stress defined by equation (15):

$$\sigma_{eff} = \sigma_y + h\varepsilon_{ep}^n, \tag{18}$$

where ε_{ep} means the effective plastic strain, h and n are the plastic material constants. For a certain plastic state characterized by σ_{eff}, the secant moduli of the matrix material can then be defined as (in plane strain case):

$$\mu_0^s = \frac{1}{(1/\mu_0) + 3[(\sigma_{eff} - \sigma_y)/h]^{1/n} / \sigma_{eff}}, \qquad \kappa_0^s = \kappa,$$

$$k_0^s = K_0 + \mu_0^s / 3, \qquad (\beta_0^s + \gamma_0^s) = 2l_m^2 \mu_0^s, \tag{19}$$

where the superscript s represents the secant quantities and K_0 denotes the bulk modulus.

The procedure for evaluating the overall nonlinear stress-strain relation by the secant modulus scheme is summarized as follows: for a given macroscopic stress $\mathbf{\Sigma}^{sym}$, at which the matrix has entered into plastic state, for a tested average effective stress of the matrix $<\sigma_{eff}>_0$ ($>\sigma_y$), the secant moduli of the matrix can be evaluated by equation (19). We consider a linear comparison composite, it has the same microstructure and fiber's property as the actual nonlinear composite, however, its matrix has the secant moduli of the actual matrix in the nonlinear composite. The compliance tensor $\mathbf{M}_C^{sym}$ of this linear comparison composite can be determined from equation (10). The average Mises stress of the micropolar matrix for the linear comparison composite can then be evaluated with help of equation (17) for a given applied load $\mathbf{\Sigma}^{sym}$. The moduli of the linear comparison composite are interpreted as the secant moduli of the actual composite. By repeating $\mathbf{\Sigma}^{sym}$, the nonlinear stress and strain relation of the composite material can then be established.

4.2 Numerical examples

In this section, some numerical calculations are performed in order to illustrate the previous theoretical formulations. An aluminum matrix containing cylindrical voids $(\mu_1 = \kappa_1 = 0, f = 0.3)$ is chosen as the sample composite material, so the interface is a free surface of the matrix material. The matrix elastic constants are $\mu_0 = \kappa_0 = 23GPa$, $k_0 = 57.5GPa$, $(\beta_0 + \gamma_0) = 2l_m^2\mu_0$. The plastic material constants $\sigma_y = 250MPa$, $h = 1.73\times10^8$, $n = 0.455$ are assumed. For convenience, we let $l_m = \delta|l_s|$. The surface properties are taken from Sharma and Ganti (2003). Two sets of the surface moduli are examined, namely,

I: $\lambda_s = 6.842\,N/m$, $\mu_s = -0.3755\,N/m$ for the surface [1 1 1];

II: $\lambda_s = 3.489\,N/m$, $\mu_s = -6.2178\,N/m$ for the surface [1 0 0].

We note that for the type II surface the compliance tensor $\mathbf{M}_s$ can be negatively definite, which may lead to some instabilities, this will be discussed in section 4.3.

Generally speaking, the interface and high-order material constants (equivalently l_m and l_s) are difficult to be determined by a direct method either through the experimental or theoretical way. However it is usually accepted that the interface length is of nanometer scale, and the nonlocal length of micropolar plasticity is of micrometer scale. To predict nonlinear overall behavior of a composite material, these constants should be at hand in advance, or be fitted from the overall experimental curves.

Under uniaxial loading, the normalized composite yielding stress as function of the void radius are shown in Figures 1 and 2 for the two types of the surface, respectively. As shown by Xun *et al.* (2004), for the micropolar effect, the predicted yielding stress always increases with decreasing void size. For the surface of the type I, nonlocal effect and surface effect are synchronized, and the yielding stress increases with the decrease of the void size. With the increase of the parameter δ, the size influential zone becomes large, and the size-effect is dominated by the nonlocal effect. For the surface of the type II, the size-dependence due to the nonlocal and surface effect are desynchronized, and a decrease of the yielding stress is predicted when the void size is smaller than a critical value, however for the case of large void size, again the nonlocal effect dominates.

Figure 3 shows the predicted yielding surfaces with the different effects, the inclusion size is set to be $R = 3l_s$ and $\delta = 1$ is used. It is found that the surface of the type I or II strengthens or weakens the composite, respectively, however nonlocal effect always predicts a strengthening effect. It is interesting to note that the interface effect leads to a significant size-dependence for a hydrostatic loading, this is beyond the capacity of micropolar theory.

The predicted macroscopic uniaxial stress-strain relations are given in Figure 4 for the surface of the type I, in the computation, $R = 3l_s$ and $\delta = 1$. It can be seen that for the surface of this type the interface effect leads to a significant strain hardening behavior, especially for small void size. Tensile stress-strain relations with various δ are illustrated in Figure 5, both nonlocal and interface effects are present, it is found that when δ exceeds 10, the micropolar effect dominates the size-dependence of the overall behavior.

4.3 Discussion on instability of the type II interface

In the computation, we found that for the surface of the type II, some instabilities can happen at certain condition. The reason is that the calculated effective moduli and second order stress moment can turn to negative values. In fact, as mentioned previously, the interface compliance tensor $\mathbf{M}_s$ can be negatively definite for the type II interface, consequently the interface elastic energy $\boldsymbol{\sigma}_s : \mathbf{M}_s : \boldsymbol{\sigma}_s$ can also be negative. This usually doesn't break the thermodynamic stability since an interface cannot exist independent of the bulk material and the total energy (bulk+interface) is positive (Shenoy, 2005). However it is possible through equation (13), that the total energy of the RVE can be negative for the type II interface when the void radius is below some critical values, especially when the volume fraction of void is high, or the bulk material experiences weakening. The similar problem was also discussed by Tian and Rajapakse (2007), where they find when the void size is below some critical values, the stress on the negative surface tends to be infinite, which is impossible in physical reality.

To examine the basic features of the instability, the overall uniaxial stress-strain relation of a Cauchy matrix with the type II surface are plotted in Figures 6 and 7 for void volume fractions $f = 10\%$ and 30% respectively. The stress-strain curves are evaluated for different void sizes, and the instable point is determined when the tangent modulus equals zero. For a certain void radius (for example $R = 8l_s$ in Figure 6), the composite material is stable at elastic state, with the development of plasticity, the matrix is weakened, the whole material turns to unstable at about $E_{11} = 0.0075$. It can be found that composites with larger voids can undergo more plastic deformation before reaching the critical points. When the void size is too small, the composite is originally unstable even in pure elastic state. Comparison of Figures 6 and 7 implies that for the same void radius, instability is reached earlier for higher volume fraction of voids. Figure 8 illustrates the comparison of the critical points for a Cauchy and a micropolar matrix. The trends are identical for the both material models, however it is more stable for the micropolar composite due to its strengthening effect.

5. Conclusions

We propose an analytical method to examine the influence of the nonlocal and interface effects on the size-dependent plastic behavior for composite materials. The nonlocal effect is considered by idealizing the matrix as a micropolar continuum model. The perturbation method for a micropolar composite with the interface effect is rigorously established, it is used to estimate the average second order stress/couple stress moment of the matrix material. The overall nonlinear behavior of a micropolar composite with the interface effect is estimated through a secant modulus scheme. The application of the presented method to composites with cylindrical voids shows that both nonlocal and interface effects have a significant influence on the size-dependent yielding surface and the strain hardening behavior. The interface effect can predict a strong size-dependence for a hydrostatic loading. When the void size tends to be large, the influence of the interface effect becomes small, then micropolar effect dominates. Finally classical prediction can be recovered for infinite void size, as expected. The interface of the type II can trigger instability of the composite, and the unstable strain depends on the void size, its volume fraction and also on the weakening of the bulk material during plastic deformation.

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Appendix

Based on the solution of a two-dimensional single inclusion embedded in a micropolar media with the interface effect, the closed-form expressions for the overall in-plane bulk and shear modulus for a cylindrical fiber reinforced composite can be obtained by MTM method:

$$\frac{k_C}{k_0} = \frac{2k_1(1+f\mu_0/k_0)+\mu_0[2(1-f)+(l_s/R)(1+f\mu_0/k_0)]}{2(1-f)k_1+2fk_0+[2+(l_s/R)(1-f)]\mu_0}$$

$$\frac{\mu_C}{\mu_0} = \frac{\zeta_0+\zeta_1 g/R+\zeta_2 g l_s/R^2+\zeta_3 l_s/R}{\zeta_0'+\zeta_1 g/R+\zeta_2 g l_s/R^2+\zeta_3' l_s/R} \quad \text{(A1)}$$

where k_i and μ_i $(i=0,1)$ are the in-plane bulk and shear moduli for the matrix and fiber, respectively. R is the radius of the fiber, $l_m^2=(\beta_0+\gamma_0)/(2\mu_0)$ and $l_s=(\lambda_s+2\mu_s)/\mu_0$ are the two intrinsic lengths due to the nonlocal and the interface effect, and $g=l_m[(\kappa_0+\mu_0)/(2\kappa_0)]^{1/2}$. Other quantities in (A1) read as follows

$$\zeta_0=\mathrm{H}_0K_2(R/g),\quad \zeta_1=\mathrm{H}_1K_1(R/g),\quad \zeta_2=\mathrm{H}_2K_1(R/g),$$

$$\zeta_3=\mathrm{H}_3K_2(R/g),\quad \zeta_0'=\mathrm{H}_4K_2(R/g),\quad \zeta_3'=\mathrm{H}_5K_2(R/g)$$

where $K_i(\bullet)$ is the ith order second type modified Bessel function and

$$\mathrm{H}_0 = -2[2\mu_1\mu_0 + k_1(\mu_1 + \mu_0)][2\mu_1\mu_0 + k_0[\mu_1(1+f) + \mu_0(1-f)]],$$

$$\mathrm{H}_1 = \frac{1}{\kappa_0 + \mu_0}[4\kappa_0(1-f)(\mu_1 - \mu_0)(k_0 + \mu_0)(2\mu_1\mu_0 + k_1(\mu_1 + \mu_0)],$$

$$\mathrm{H}_2 = \frac{1}{\kappa_0 + \mu_0}[2\kappa_0\mu_0(1-f)(k_0 + \mu_0)[k_1(2\mu_1 - \mu_0) + \mu_1(3\mu_1 - 2\mu_0)]],$$

$$\mathrm{H}_3 = -\mu_0[\mu_1[2\mu_0(3\mu_1 + \mu_0) + k_0[3(1+f)\mu_1 + 2(2-f)\mu_0]] + k_1[\mu_0(4\mu_1 + \mu_0) + k_0[2(1+f)\mu_1 + (2-f)\mu_0]]],$$

$$\mathrm{H}_4 = \mathrm{H}_0 \frac{2\mu_0(\mu_1 - f\mu_1 + f\mu_0) + k_0(\mu_1 - f\mu_1 + \mu_0 + f\mu_0)}{2\mu_1\mu_0 + k_0(\mu_1 + f\mu_1 + \mu_0 - f\mu_0)},$$

$$\mathrm{H}_5 = -\mu_0[(1-f)k_0\mu_1(2k_0 + 3\mu_1) + \mu_0[(2+f)k_1k_0 + 4(1-f)k_1\mu_1 + 2\mu_1[(2+f)k_0 + 3(1-f)\mu_1]] + (1+2f)(k_1 + 2\mu_1)\mu_0^2].$$

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Figure captions

Fig.1 Normalized overall yielding stress as function of void radius for different δ values, surface of the type I, uniaxial tension, $f = 30\%$.

Fig.2 Normalized overall yielding stress as function of void radius for different δ values, surface of the type II, uniaxial tension, $f = 30\%$.

Fig.3 Comparison of macroscopic yielding surfaces predicted with or without micropolar and interfacial effect, $R = 3l_s$, $\delta = 1$, $f = 30\%$.

Fig.4 Overall uniaxial stress-strain curves predicted with or without micropolar and surface effect of the type I, $R = 3l_s$, $\delta = 10$, $f = 30\%$.

Fig.5 Overall uniaxial stress-strain curves for different δ values, both micropolar and surface effect of the type I are present, $R = 3l_s$, $f = 30\%$.

Fig.6 Critical unstable points versus void radius for Cauchy matrix and surface effect of the type II, $f = 10\%$.

Fig.7 Critical unstable points versus void radius for Cauchy matrix and surface effect of the type II, $f = 30\%$.

Fig.8 Comparison of critical unstable points for Cauchy and micropolar matrix with surface of the type II, $\delta = 10$, $f = 30\%$.

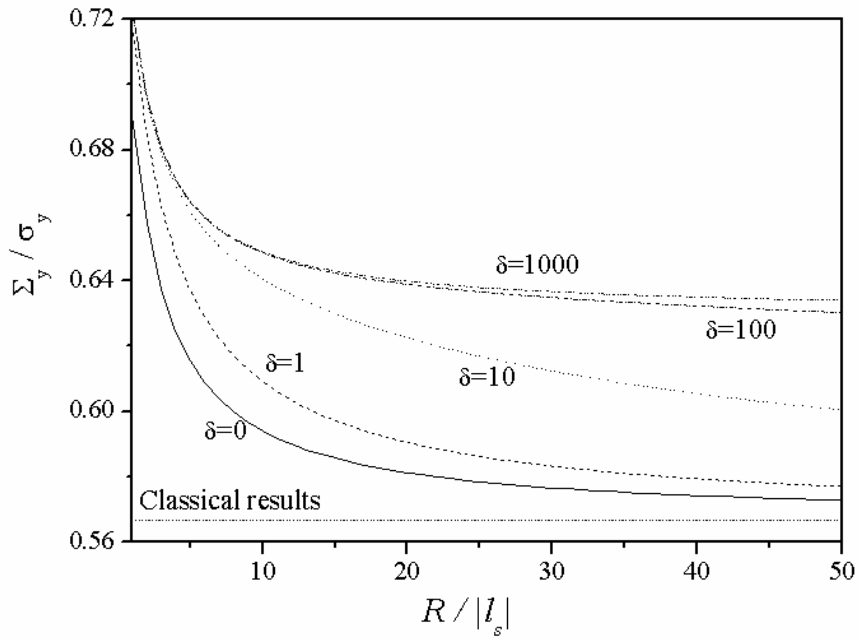

0.72
0.68
0.64
0.60
0.56
Σ_y / σ_y
δ=1000
δ=100
δ=10
δ=1
δ=0
Classical results
10
20
30
40
50
$R / |l_s|$

Fig.1

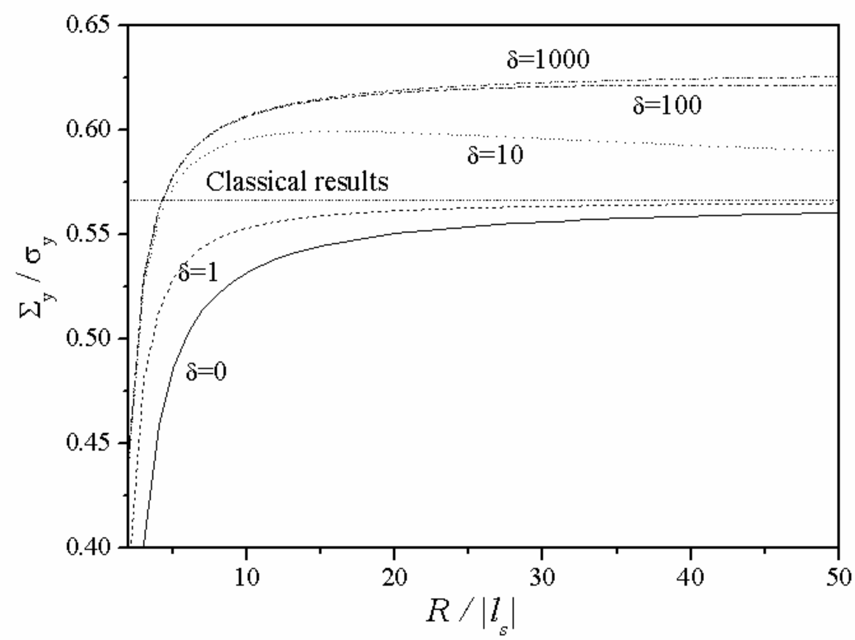
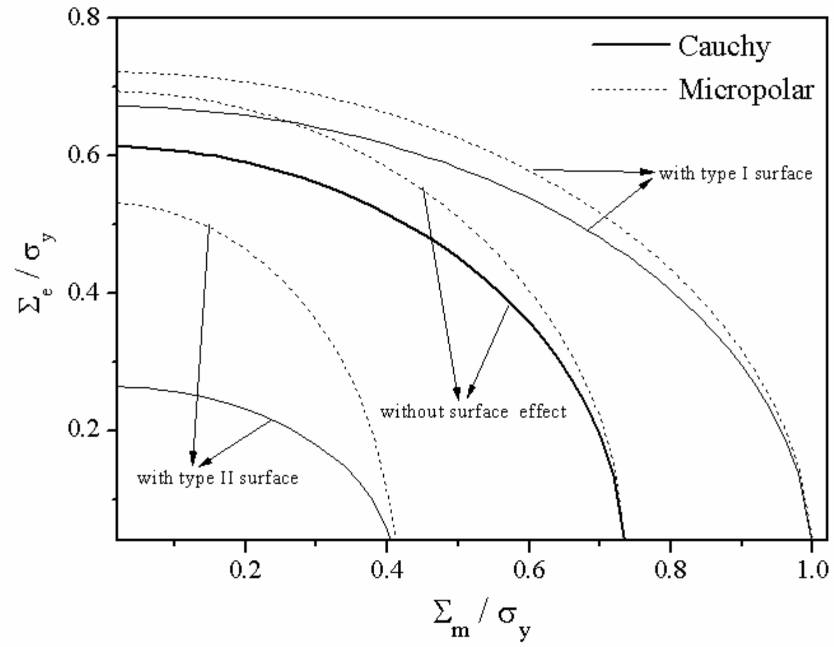
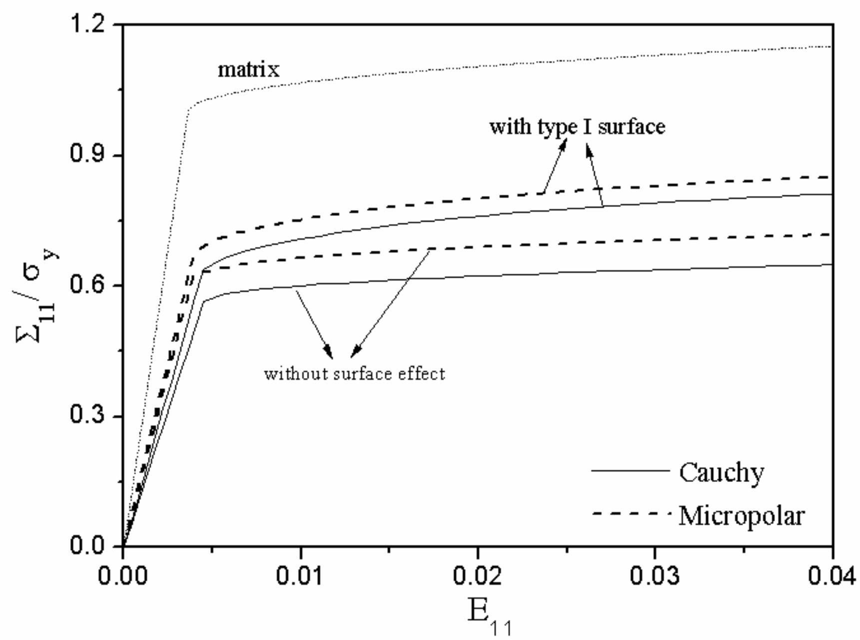

0.65
0.60
0.55
0.50
0.45
0.40
Σ_y / σ_y
δ=1000
δ=100
δ=10
Classical results
δ=1
δ=0
10
20
30
40
50
$R / |l_s|$

Fig.2



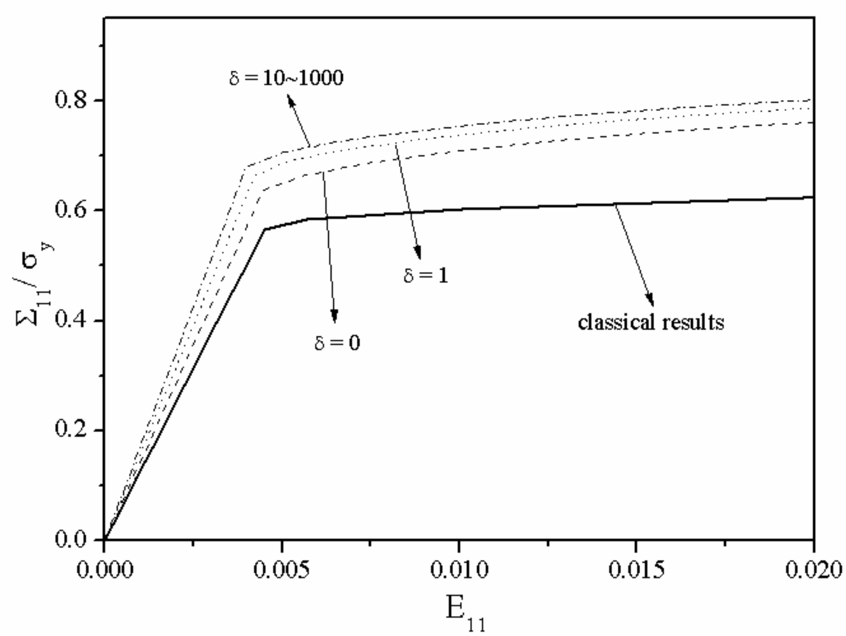
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0
Cauchy
Micropolar
with type I surface
without surface effect
with type II surface
Σ_e / σ_y
Σ_m / σ_y

Fig. 3



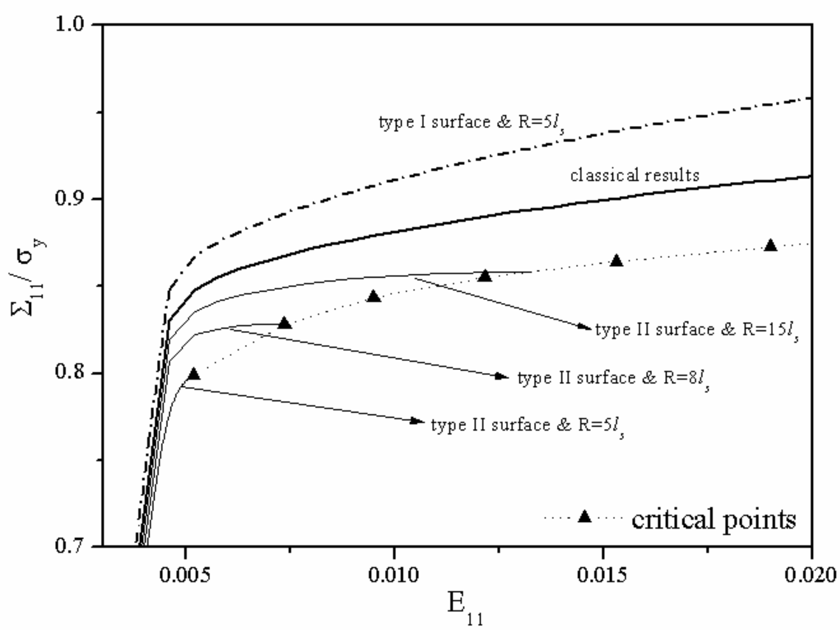
1.2
0.9
0.6
0.3
0.0
0.00
0.01
0.02
0.03
0.04
matrix
with type I surface
without surface effect
Cauchy
Micropolar
Σ_{11} / σ_y
E_{11}

Fig.4



δ = 10~1000
δ = 1
δ = 0
classical results
0.0
0.2
0.4
0.6
0.8
0.000
0.005
0.010
0.015
0.020
Σ11/σy
E11

Fig.5



type I surface & R=5ls
classical results
type II surface & R=15ls
type II surface & R=8ls
type II surface & R=5ls
critical points
0.7
0.8
0.9
1.0
0.005
0.010
0.015
0.020
Σ11/σy
E11

Fig.6

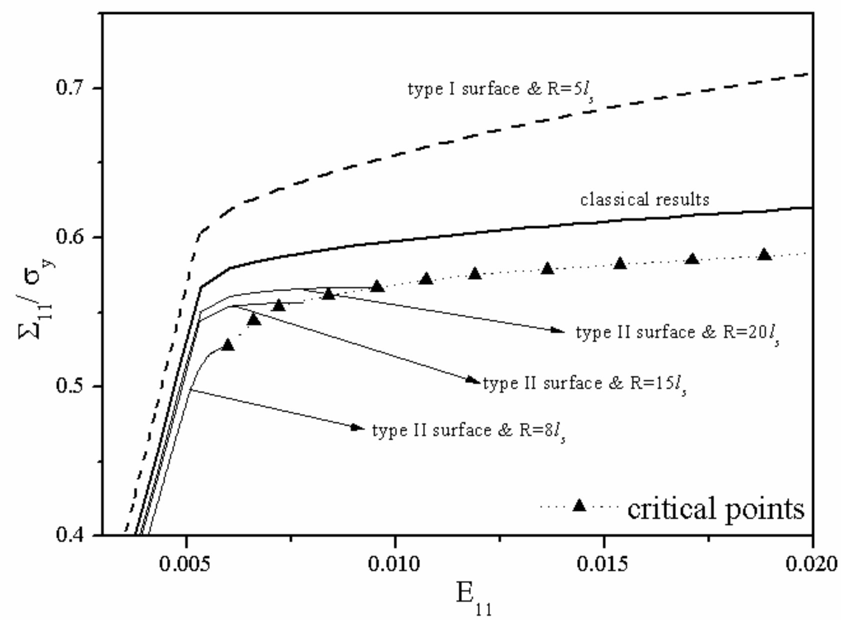

type I surface & R=5l_s
classical results
type II surface & R=20l_s
type II surface & R=15l_s
type II surface & R=8l_s
critical points
Σ_{11}/σ_y
E_{11}
0.4
0.5
0.6
0.7
0.005
0.010
0.015
0.020

Fig.7

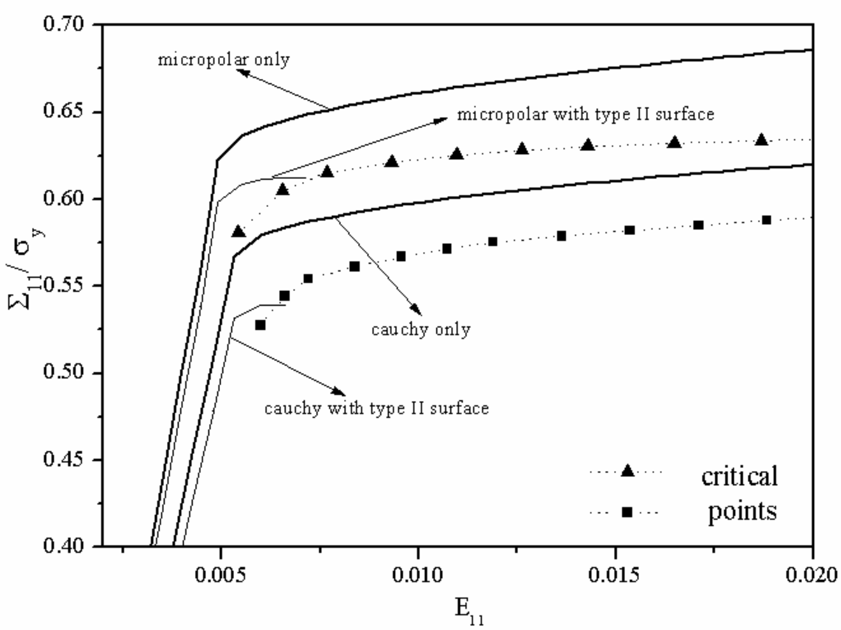

micropolar only
micropolar with type II surface
cauchy only
cauchy with type II surface
critical
points
Σ_{11}/σ_y
E_{11}
0.40
0.45
0.50
0.55
0.60
0.65
0.70
0.005
0.010
0.015
0.020

Fig.8