ELSEVIER

Composites: Part A 33 (2002) 717–724

composites
Part A: applied science and manufacturing

www.elsevier.com/locate/compositesa

Thermal expansion of composites with shape memory alloy inclusions and elastic matrix

G.K. Hu[a,*], Q.P. Sun[a,b]

[a]Department of Mechanical Engineering, The Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong, People's Republic of China
[b]Department of Engineering Mechanics, Tsinghua University, Beijing 100084, People's Republic of China

Received 3 July 2000; revised 7 May 2001; accepted 3 January 2002

Abstract

Incorporation of shape memory alloy (SMA) inclusions into a continue matrix can make composites with various thermal dilatation behavior, and this depends sensitively on the microstructural parameters of the composite. A micro-mechanical method is proposed to relate quantitatively the overall thermal dilatation with the microstructures and the transformation characteristics of SMA materials. Composites with aligned and with two populations of perpendicularly oriented SMA inclusions are analyzed in detail in this paper. It is found that the composite with SMA fibers can have a large transformation temperature range during the heating process, and a linear shrinkage may take place during the cooling because of the large difference between the austenite finish and martensite start temperatures of the composite. Design aspects to minimize the overall thermal dilatation during a full thermal cycle are also discussed.

Keywords: A. Metal-matrix composites; B. Mechanical properties; B. Microstructure

1. Introduction

Under a thermo-mechanical loading, adequately pre-strained shape memory alloys (SMAs) can memorize their original form through reversible martensitic phase transformation. This specific property of SMAs has promising applications in active control and smart structures. Thermo-mechanical behavior of SMA materials has received intensive study in recent years, and a number of models have been proposed to characterize their thermo-mechanical responses. They can be roughly classified as: micro-mechanical models [1–3] and phenomenological models [4–6]. The SMA inclusions can be placed into a continue matrix to form a composite, by properly designing the distribution and the form of the SMA inclusions, the composite may have the desired and sometime unique overall response under a thermo-mechanical loading. Researches on this subject have now received increasing attention [7]. For example, a systematic and comprehensive study on composites with SMA inclusions is performed by Lagoudas et al. [8], they utilize the Mori–Tanaka's micro-mechanical model to take into account the interaction of the SMA inclusions. More recently, with the aid of incremental self-consistent method, Cherkaoui et al. [9] and Song et al. [10] extended the analyses by including the plastic effect of the continue matrix.

In this paper, we will focus on the overall thermal expansion of composites with SMA inclusions. The basic idea is as follows: SMA inclusions are pre-strained to have certain martensitic transformation and they are then mixed with continue elastic matrix to form a composite. Under a subsequent temperature variation such as heating, the different thermal expansions between the matrix and the SMA inclusions will impose a stress on the SMA inclusions, and this stress together with the temperature may trigger the reverse transformation in the SMA inclusions. The shrinkage of the SMA inclusions, due to the reverse transformation, will partially or totally compensate the expansion of the matrix, leading to a composite with a specified overall thermal expansion behavior during the heating and the subsequent cooling process. The main objective of this paper is to elucidate and model these mechanisms and to discuss some practical design aspects.

The paper is organized as follows: in Section 2, the stress in the SMA inclusions and the overall expansion of the

* Corresponding author. Address: Department of Applied Mechanics, Beijing Institute of Technology, 100081, Beijing, China. Fax: +81-727-51-9970.
E-mail address: hugeng@public.bta.net.cn (G.K. Hu).

1359-835X/02/$ - see front matter
PII: S1359-835X(02)00009-X

composite are derived analytically for a thermal loading. A phase transformation equation for the SMA inclusions is introduced in Section 3. Detailed analyses for composites with aligned SMA inclusions and with two perpendicularly oriented populations of SMA inclusions are presented in Section 4. Some practical design aspects for minimizing the overall thermal expansion of the composite during heating and the overall contraction during cooling are discussed in Section 5.

2. Theoretical formulation

The composite considered consists of an elastic matrix and two populations of SMA inclusions. The SMA inclusions are arranged perpendicularly as shown in Fig. 1. The constituents are assumed to be isotropic, and each population of the inclusions has the same aspect ratio. Even though SMA shows anisotropic properties at single crystal or single grain levels, the SMAs used in composite are normally polycrystals and at this level it can be considered to be isotropic macroscopically. The stiffness (compliance) tensor and the thermal dilatation of the SMA inclusions are denoted by $\mathbf{L}_\mathrm{f}(\mathbf{M}_\mathrm{f})$ and $\underline{\underline{\alpha}}_\mathrm{f}(\underline{\underline{\alpha}}_\mathrm{f} = \alpha_\mathrm{f}\underline{\underline{I}})$, respectively, and the corresponding stiffness (compliance) and the thermal dilatation for the continue matrix are $\mathbf{L}_\mathrm{m}(\mathbf{M}_\mathrm{m})$ and $\underline{\underline{\alpha}}_\mathrm{m}(\underline{\underline{\alpha}}_\mathrm{m} = \alpha_\mathrm{m}\underline{\underline{I}})$. In this paper, $\underline{\underline{(\bullet)}}$ denotes a second order tensor, and the bold letter is a fourth order tensor, $\underline{\underline{I}}$ is the second order unit tensor.

The micro-mechanical analyses for SMA materials show that the stiffness tensor for the SMA materials follows well the mixture law between the martensitic phase and the parent austenite during the transformation [2]. However, for the sake of simplification, it is considered as a constant in this paper. The difference in stiffness can be taken into account by a two scale micro-mechanical method, but this will substantially complicate the quantitative treatment. In the following, we will derive the overall thermal expansion of the composite by employing the Mori–Tanaka's micro-mechanical model that takes into account the interaction between the inclusions. For the considered composite, it is shown that this method is free from the asymmetric prediction for the overall stiffness tensor [11]. Here we neglect the latent heat effect of the transformation and isothermal condition is assumed. The composite is subjected only to a uniform temperature change ΔT, the thermal and transformation induced strains are simulated by an eigenstrain $\underline{\underline{\epsilon}}_\mathrm{T}^0$ in the inclusions: $\underline{\underline{\epsilon}}_\mathrm{T}^0 = \underline{\underline{\epsilon}}_\mathrm{t} + \underline{\underline{\epsilon}}_\mathrm{tr}^0$ for the inclusions perpendicularly aligned (shown in Fig. 1), where $\underline{\underline{\epsilon}}_\mathrm{t} = (\underline{\underline{\alpha}}_\mathrm{f} - \underline{\underline{\alpha}}_\mathrm{m})\Delta T$ is the mismatch strain induced by ΔT and $\underline{\underline{\epsilon}}_\mathrm{tr}^0$ is the eigenstrain related to the transformation of the SMA inclusions; for the inclusions horizontally aligned, it is denoted by $\underline{\underline{\epsilon}}_\mathrm{T}^{90} = \underline{\underline{\epsilon}}_\mathrm{t} + \underline{\underline{\epsilon}}_\mathrm{tr}^{90}$.

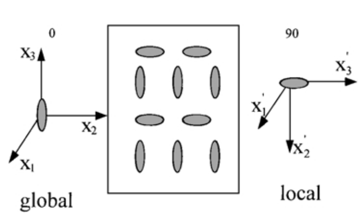

Fig. 1. Sketch of composite microstructures.

According to Mori–Tanaka's method, the estimated overall thermal expansion of the composite has the following final form (the readers are advised to refer Appendix A for details):

$$\underline{\underline{E}} = \underline{\underline{\alpha}}_\mathrm{c}\Delta T + f^0\mathbf{R}_1 : \underline{\underline{\epsilon}}_\mathrm{tr}^0 + f^{90}\mathbf{R}_2 : \underline{\underline{\epsilon}}_\mathrm{tr}^{90} \tag{1}$$

where $\underline{\underline{\alpha}}_\mathrm{c}$ is the overall thermal coefficient of the composite given by:

$$\begin{aligned}
&\underline{\underline{\alpha}}_\mathrm{c} = \underline{\underline{\alpha}}_\mathrm{m} + (f^0\mathbf{R}_1 + f^{90}\mathbf{R}_2) : (\underline{\underline{\alpha}}_\mathrm{f} - \underline{\underline{\alpha}}_\mathrm{m}) \\
&\mathbf{R}_1 = \mathbf{I} + f_\mathrm{m}\mathbf{T}(\mathbf{Q} - \mathbf{I}), \qquad \mathbf{R}_2 = \mathbf{I} + f_\mathrm{m}\mathbf{T}(\bar{\mathbf{Q}} - \mathbf{I}), \\
&\mathbf{T} = [f_\mathrm{m}\mathbf{I} + f^0\mathbf{Q} + f^{90}\bar{\mathbf{Q}}]^{-1}, \\
&\mathbf{Q} = -[(\mathbf{M}_\mathrm{f}\mathbf{L}_\mathrm{m} - \mathbf{I})(\mathbf{S}^0 - \mathbf{I}) - \mathbf{I}]^{-1}, \\
&\text{and } \bar{\mathbf{Q}} = -\mathbf{U}^{-1}[(\mathbf{M}_\mathrm{f}\mathbf{L}_\mathrm{m} - \mathbf{I})(\mathbf{S}^{90} - \mathbf{I}) - \mathbf{I}]^{-1}\mathbf{U}
\end{aligned} \tag{2}$$

f_m is the volume fraction of the matrix and f^0, f^{90} are those for the SMA inclusions with the perpendicular and horizontal orientations, respectively. $\mathbf{I}$ is an identity tensor of rank four, and $[\bullet]^{-1}$ means the inverse of the said quantity. $\mathbf{U}$ is the transformation matrix, which relates the stress or strain from the local system to the global one (in Voigt notation for stress or strain) by $\underline{\underline{\sigma}} = \mathbf{U} : \underline{\underline{\sigma}}$. For the considered problem as shown in Fig. 1, the transformation matrix is given in Appendix A. $\mathbf{S}^0, \mathbf{S}^{90}$ are the Eshelby tensors of the perpendicularly and horizontally oriented SMA inclusions, their analytical expressions can be found in the book of Mura [12].

The stresses in the inclusion, which, together with temperature are the key factors determining the extent of the martensitic transformation, can be expressed as:

$$\begin{aligned}
\underline{\underline{\sigma}}^0 = {} & (\mathbf{M}_\mathrm{f} - \mathbf{M}_\mathrm{m})^{-1}[(\mathbf{I} - f^0\mathbf{Q}\mathbf{T})(\mathbf{Q} - \mathbf{I}) \\
& : \underline{\underline{\epsilon}}_\mathrm{T}^0 - f^{90}\mathbf{Q}\mathbf{T}(\overline{\mathbf{Q}} - \mathbf{I}) : \underline{\underline{\epsilon}}_\mathrm{T}^{90}]
\end{aligned} \tag{3}$$

$$\begin{aligned}
\underline{\underline{\sigma}}^{90} = {} & (\mathbf{M}_\mathrm{f} - \mathbf{M}_\mathrm{m})^{-1}[(\mathbf{I} - f^{90}\overline{\mathbf{Q}}\mathbf{T})(\bar{\mathbf{Q}} - \mathbf{I}) \\
& : \underline{\underline{\epsilon}}_\mathrm{T}^{90} - f^0\bar{\mathbf{Q}}\mathbf{T}(\mathbf{Q} - \mathbf{I}) : \underline{\underline{\epsilon}}_\mathrm{T}^0]
\end{aligned} \tag{4}$$

When $f^0 = f^{90} = 0$, Eqs. (3) and (4) gives the stress for an inclusion embedded in the infinite matrix. Although the formulation here is given for two populations of SMA inclusions, it can be easily extended to any prescribed textures. The variation of temperature ΔT induces a thermal stress in the SMA inclusions, which can be estimated by Eqs. (3) and (4). This stress together with the temperature controls the forward and reverse transformations of the SMA inclusions. At the same time the transformation strains in turn influence the amplitude of the stresses in the inclusions through again

Eqs. (3) and (4), so this is a coupled iterative process. To this end, the transformation characteristic of the SMA materials has to be provided, and this will be briefly outlined in the following.

3. Transformation law for the shape memory alloys inclusions

In this paper, the three-dimensional constitutive equation proposed by Lagoudas et al. [8] will be adopted. The thermal compensate mechanisms can be described thus: the SMA inclusions are pre-loaded to have a forward transformation before they are embedded into a continue matrix to form a composite. This state is the start of our analysis and the composite is assumed to be stress free at T_0. When the composite is subjected to a temperature rise (heating process), the matrix will impose a tensile stress on the SMA inclusions. When T reaches a critical value, the reverse transformation (martensite → austenite) will happen. If the microstructures of the composite are properly designed, the shrinkage due to the reverse transformation strain of the SMA inclusions will fully or partially compensate the thermal dilatation of the matrix. During the subsequent cooling process, the stress and the temperature on the SMA inclusions may induce the forward transformation (austenite → martensite, due to the tensile stress in the SMA caused by previous reverse transformation), the reverse compensate mechanism prevails (expansion of the SMA inclusions and shrinkage of the matrix).

Following Lagoudas et al. [8], during the reverse transformation, the incremental transformation strain can be written as:

$$\dot{\underline{\underline{\epsilon}}}_{\mathrm{mtr}} = \dot{\zeta}\gamma\frac{\underline{\underline{\epsilon}}_{\mathrm{mtr}}}{\bar{\epsilon}_{\mathrm{mtr}}} \tag{5}$$

where $\underline{\underline{\epsilon}}_{\mathrm{mtr}}$ is the total transformation strain, it is related to the eigenstrain by:

$$\underline{\underline{\epsilon}}_{\mathrm{T}} = \underline{\underline{\epsilon}}_{\mathrm{t}} + \underline{\underline{\epsilon}}_{\mathrm{tr}} = \underline{\underline{\epsilon}}_{\mathrm{t}} + \underline{\underline{\epsilon}}_{\mathrm{mtr}} - \gamma\underline{\underline{\Lambda}}, \qquad \bar{\epsilon}_{\mathrm{mtr}} = \sqrt{2/3\underline{\underline{\epsilon}}_{\mathrm{mtr}} : \underline{\underline{\epsilon}}_{\mathrm{mtr}}}$$

and γ is the total equivalent transformation strain of the forward transformation during the pre-loading process (here it is assumed that the transformation is complete and that γ is a material constant). In the coordinate system where X_3 coincides with the symmetric axis of the inclusions (which is also the direction of the pre-loading), $\underline{\underline{\Lambda}}$ has the following form: $\Lambda_{11} = \Lambda_{22} = -1/2$, $\Lambda_{33} = 1$ and the other components are zero. ζ is the volume fraction of the martensitic phase in SMA, and is related to the stress and the temperatures by [9]:

$$\zeta = \exp[a^{\mathrm{A}}(A^{0\mathrm{s}} - T) + b^{\mathrm{A}}\bar{\sigma}] \tag{6}$$

where $\bar{\sigma}$ is the Von Mises effective stress in the SMA inclusions, and

$$a^{\mathrm{A}} = \frac{\ln(0.01)}{A^{0\mathrm{s}} - A^{0\mathrm{f}}}, \qquad b^{\mathrm{A}} = \frac{a^{\mathrm{A}}}{c^{\mathrm{A}}},$$

$A^{0\mathrm{s}}, A^{0\mathrm{f}}$ are, respectively, the austenite start and finish temperatures for the SMA material under stress-free state.

During the forward transformation, the volume fraction of the martensitic phase now is written as:

$$\zeta = 1 - \exp[a^{\mathrm{M}}(M^{0\mathrm{s}} - T) + b^{\mathrm{M}}\bar{\sigma}] \tag{7}$$

where $a^{\mathrm{M}} = \ln(0.01)/(M^{0\mathrm{s}} - M^{0\mathrm{f}})$, $b^{\mathrm{M}} = a^{\mathrm{M}}/c^{\mathrm{M}}$; and $M^{0\mathrm{s}}$, $M^{0\mathrm{f}}$ are the martensite start and finish temperatures under stress-free state, $c^{\mathrm{A}}, c^{\mathrm{M}}$ are the material constants.

Due to the temperature variation, a stress will be imposed on the SMA inclusions, macroscopically the austenite (martensite) start and finish temperatures for the composite will be shifted to A^{s} and $A^{\mathrm{f}}(M^{\mathrm{s}}, M^{\mathrm{f}})$. So the earlier transformation relation is applied only for the temperature range $A^{\mathrm{s}} \le T \le A^{\mathrm{f}}$ for the reverse and $M^{\mathrm{f}} \le T \le M^{\mathrm{s}}$ for the forward transformation of the composite. For the temperature out of this range, the SMA inclusions can be considered as elastic inclusions with the fixed transformation strains only.

With the help of the transformation law for the SMA material outlined previously, and also the applied stresses given by Eqs. (3) and (4), the transformation strain as function of temperature can be determined. The overall expansion of the composite can be calculated with the help of Eqs. (1) and (2). Two special cases will be studied in detail in the following: one is a composite with the aligned SMA inclusions; the other is a composite with perpendicularly oriented SMA inclusions.

4. Applications

4.1. Composite with aligned shape memory alloys inclusions

For the case of one population of the SMA inclusions, by setting $f^{90} = 0$ and $f^{0} = f$, we get from Eqs. (1)–(4)

$$\underline{\underline{E}} = [\underline{\underline{\alpha}}_{\mathrm{m}} + f\mathbf{R}_1 : (\underline{\underline{\alpha}}_{\mathrm{f}} - \underline{\underline{\alpha}}_{\mathrm{m}})]\Delta T + f\mathbf{R}_1 : (\underline{\underline{\epsilon}}^{0}_{\mathrm{mtr}} - \gamma\underline{\underline{\Lambda}}) \tag{8}$$

$$\underline{\underline{\sigma}}^{0} = [\mathbf{M}_{\mathrm{f}} - \mathbf{M}_{\mathrm{m}}]^{-1}(\mathbf{R}_1 - \mathbf{I}) : [(\underline{\underline{\alpha}}_{\mathrm{f}} - \underline{\underline{\alpha}}_{\mathrm{m}})\Delta T + (\underline{\underline{\epsilon}}^{0}_{\mathrm{mtr}} - \gamma\underline{\underline{\Lambda}})] \tag{9}$$

Suppose that $\alpha_{\mathrm{m}} > \alpha_{\mathrm{f}}$, so during a temperature rise ($\Delta T > 0$), a tensile stress σ^{0}_{33} will be generated in the inclusion's direction. For the considered symmetric problem, the stress components in the inclusions are: $\sigma^{0}_{11} = \sigma^{0}_{22}, \sigma^{0}_{33}$ and the transformation strain components are: $\epsilon^{0}_{\mathrm{mtr11}} = \epsilon^{0}_{\mathrm{mtr22}} = -1/2\epsilon^{0}_{\mathrm{mtr33}}$. This leads to the equivalent stress $\bar{\sigma}^{0} = \sigma^{0}_{33} - \sigma^{0}_{11}$ and $\underline{\underline{\epsilon}}^{0}_{\mathrm{mtr}}/\bar{\epsilon}^{0}_{\mathrm{mtr}} = \underline{\underline{\Lambda}}$. Together with the help of Eqs. (5) and (6), the governing differential equation for determining $\epsilon^{0}_{\mathrm{mtr33}}$ is:

$$\frac{\mathrm{d}\epsilon^{0}_{\mathrm{mtr33}}}{\mathrm{d}T} = \frac{-a^{\mathrm{A}} + b^{\mathrm{A}}h_1(\alpha_{\mathrm{f}} - \alpha_{\mathrm{m}})}{\exp[-a^{\mathrm{A}}(A^{0\mathrm{s}} - T) - b^{\mathrm{A}}(h_1(\alpha_{\mathrm{f}} - \alpha_{\mathrm{m}})(T - T_0) + h_2(\epsilon^{0}_{\mathrm{mtr33}} - \gamma))]/\gamma - b^{\mathrm{A}}h_2} \tag{10}$$

with the initial conditions $T = A^s$ and $\epsilon^0_{mtr33} = \gamma$. With the help of Eqs. (6) and (9), the austenite start and finish temperatures of the composite can be determined as:

$$A^s = T_0 + \frac{A^{0s} - T_0}{1 - h_1(\alpha_f - \alpha_m)/c^A}, \qquad A^f = T_0 + \frac{A^{0s} - T_0 + 4.5/a^A - h_2\gamma/c^A}{1 - h_1(\alpha_f - \alpha_m)/c^A} \tag{11}$$

where austenite finish temperature is calculated at $\zeta = 0.01$. During the forward transformation (cooling process), the transformation strain is determined by:

$$\frac{d\epsilon^0_{mtr33}}{dT} = \frac{-a^M + b^M h_1(\alpha_f - \alpha_m)}{-\exp[-a^M(M^{0s} - T) - b^M(h_1(\alpha_f - \alpha_m)(T - T_0) + h_2(\epsilon^0_{mtr33} - \gamma))]/\gamma - b^M h_2} \tag{12}$$

with the initial condition $T = M^f, \epsilon^0_{mtr33} = \gamma$. During the decrease of the temperature, the martensite start and finish temperatures are given by:

$$M^s = T_0 + \frac{M^{0s} - T_0 - h_2\gamma/c^M}{1 - h_1(\alpha_f - \alpha_m)/c^M}, \qquad M^f = T_0 + \frac{M^{0s} - T_0 + 4.5/a^M}{1 - h_1(\alpha_f - \alpha_m)/c^M} \tag{13}$$

The influence of the microstructures on the transformation is characterized by two parameters $h_1$and h_2, which are given in Appendix A for general ellipsoidal shape of the inclusions. In the following, only the expressions of h_1, h_2 for some special shapes of the inclusions are listed

(a) Long fiber

$$h_1 = -\frac{9b(1-f)\kappa_m\mu_m\mu_f}{[(1 + a(1-f))(4 + 3b(1-f))\mu_m + (3 + 3b(1-f))\kappa_m](\mu_m - \mu_f)}$$

$$h_2 = -\frac{9b(1-f)\mu_m\mu_f[\kappa_m + (1 + a(1-f))\mu_m]}{[(1 + a(1-f))(4 + 3b(1-f))\mu_m + (3 + 3b(1-f))\kappa_m](\mu_m - \mu_f)}$$

(b) Sphere

$$h_1 = 0$$

$$h_2 = -\frac{3b(1-f)(9\kappa_m + 8\mu_m)\mu_m\mu_f}{[3(5 + 3b(1-f))\kappa_m + 4(5 + 2b(1-f))\mu_m](\mu_m - \mu_f)}$$

(c) Penny shape

$$h_1 = \frac{18b(1-f)\kappa_m\mu_m\mu_f}{[3(1 + b(1-f))\kappa_m + 4(1 + a(1-f))\mu_m](\mu_m - \mu_f)}$$

$$h_2 = -h_1/2$$

where $a = \kappa_m/\kappa_f - 1,\ b = \mu_m/\mu_f - 1,\ \kappa_m, \mu_m, \kappa_f, \mu_f$ are the bulk and shear moduli for the matrix and inclusions, respectively.

Once the transformation strain as function of temperature is determined, the overall thermal expansion can be written as (from Eq. (8)):

For the heating process ($T_0 \to A^f$):

$$E_{11} = [\alpha_m + fg_1(\alpha_f - \alpha_m)](T - T_0)$$
$$E_{33} = [\alpha_m + fg_3(\alpha_f - \alpha_m)](T - T_0) \text{ for } T_0 \le T \le A^s \tag{14}$$

$$E_{11} = [\alpha_m + fg_1(\alpha_f - \alpha_m)](T - T_0) + g_2\left(\epsilon^0_{mtr33} - \gamma\right)$$
$$E_{33} = [\alpha_m + fg_3(\alpha_f - \alpha_m)](T - T_0) + g_4\left(\epsilon^0_{mtr33} - \gamma\right) \tag{15}$$
for $A^s \le T \le A^f$

and for the cooling process ($A^f \to T_0$)

$$E_{11} = [\alpha_m + fg_1(\alpha_f - \alpha_m)](T - T_0) - g_2\gamma$$
$$E_{33} = [\alpha_m + fg_3(\alpha_f - \alpha_m)](T - T_0) - g_4\gamma \text{ for } M^s \le T \le A^f \tag{16}$$

and

$$E_{11} = [\alpha_m + fg_1(\alpha_f - \alpha_m)](T - T_0) + g_2\left(\epsilon^0_{mtr33} - \gamma\right)$$
$$E_{33} = [\alpha_m + fg_3(\alpha_f - \alpha_m)](T - T_0) + g_4\left(\epsilon^0_{mtr33} - \gamma\right)$$
for $T_0 \le T \le M^s$ (17)

It is shown that usually $T_0 > M^f$, this means the martensitic phase cannot be recovered completely during a thermal cycle. So Eq. (17) starts from T_0. The coefficients g_1, g_2, g_3, g_4 depend on the microstructures and their general forms are also listed in Appendix A. For special shapes of the SMA inclusions they are:

(a) Long fiber

$$g_1 = \frac{[3(2 + 3b(1-f))\kappa_m + 2(4 + 3b(1-f))\mu_m]}{2[(1 + a(1-f))(4 + 3b(1-f))\mu_m + (3 + 3b(1-f))\kappa_m]}$$

$$g_2 = -\frac{[3\kappa_m + 2(2 + 3a(1-f))\mu_m]}{2[(1 + a(1-f))(4 + 3b(1-f))\mu_m + (3 + 3b(1-f))\kappa_m]}$$

$$g_3 = \frac{[3\kappa_m + (4 + 3b(1-f))\mu_m]}{[(1 + a(1-f))(4 + 3b(1-f))\mu_m + (3 + 3b(1-f))\kappa_m]}$$

$$g_4 = \frac{[3\kappa_m + (4 + 3a(1-f))\mu_m]}{[(1 + a(1-f))(4 + 3b(1-f))\mu_m + (3 + 3b(1-f))\kappa_m]}$$

(b) Sphere inclusion

$$g_1 = \frac{3\kappa_m + 4\mu_m}{3\kappa_m + 4(1 + a(1-f))\mu_m}$$

$$g_2 = -\frac{5(3\kappa_m + 4\mu_m)}{2[3(5 + 3b(1-f))\kappa_m + 4(5 + 2b(1-f))\mu_m]}$$

$$g_3 = g_1 \qquad g_4 = -2g_2$$

(c) Penny shape

$$g_1 = \frac{3\kappa_m + 4\mu_m}{[3(1 + b(1-f))\kappa_m + 4(1 + a(1-f))\mu_m]}$$

$$g_2 = -g_1/2$$

$$g_3 = \frac{3(1 + 3b(1-f))\kappa_m + 4\mu_m}{[3(1 + b(1-f))\kappa_m + 4(1 + a(1-f))\mu_m]}$$

$$g_4 = \frac{3\kappa_m + 2(2 + 3a(1-f))\mu_m}{[3(1 + b(1-f))\kappa_m + 4(1 + a(1-f))\mu_m]}$$

It is of interest to examine the model for a practical composite system. In the following the SMA composite used by Lagoudas et al. [8] are studied in this paper:

For the NiTi SMA inclusions:

$$E_1 = 20\,000\ \text{MPa},\quad \nu_1 = 0.33,\quad \alpha_f = 8\times 10^{-6}/°\text{C},$$

$$M^{0s} = 23\ °\text{C},\quad M^{0f} = 5\ °\text{C},\quad A^{0s} = 29\ °\text{C},\quad A^{0f} = 51\ °\text{C},$$

$$c^A = 4.5\ \text{MPa}/°\text{C}\quad c^M = 11.3\ \text{MPa}/°\text{C},\quad \gamma = 4.5\%$$

For the polymer matrix:

$$E_0 = 2000\ \text{MPa},\quad \nu_0 = 0.45,\quad \alpha_m = 75\times 10^{-6}/°\text{C and}$$

$$T_0 = 24\ °\text{C}$$

The longitudinal thermal expansion of the SMA composite as a function of temperature is shown in Fig. 2 for a thermal cycle $T_0 \rightarrow A^s \rightarrow A^f \rightarrow T_0$. In the heating process, the addition of a small quantity of the SMA inclusions can significantly reduce the overall thermal strain of the composites compared to the pure matrix. By choosing an adequate volume fraction of the SMA inclusions, the overall thermal expansion can be controlled to be very small (Fig. 2 for $f = 3.5\%$ of SMA fibers). The tendency of the longitudinal thermal expansion as function of temperature is not altered significantly when the inclusion's shape changes from long fiber to penny-shape disk, however, the shape of the SMA inclusions has a significant influence on the austenite start and finish temperatures of the composite, which is showed in Fig. 3. For the composites with the SMA long fibers, the transformation takes place over a large temperature range. In order to achieve small overall thermal expansion of the composite $A^f - A^s$ must be large enough to satisfy working temperature range. In the cooling process, as shown in Fig. 2, there is large linear shrinkage of the composite due to the large difference between A^f and M^s. So in order to minimize the overall thermal expansion, $A^f - M^s$ must be kept small. The variation of $A^f - M^s$ as function of c^M/c^A($c^A = 4.5$ MPa/°C) is shown in Fig. 4 for

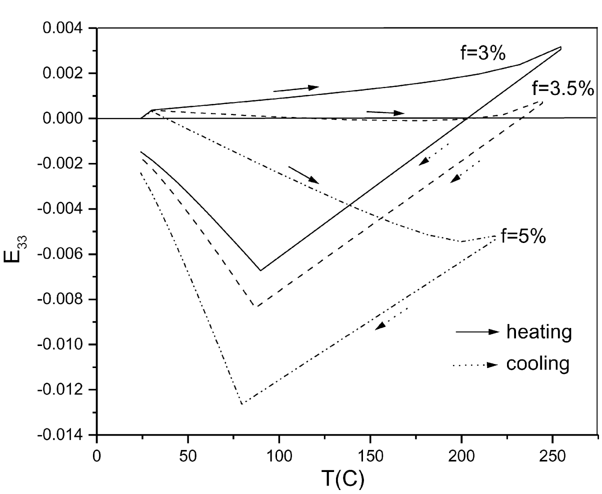

Fig. 2. Longitudinal expansion of a composite with aligned SMA fibers during a full temperature cycle ($T_0 \rightarrow A^f \rightarrow T_0$).

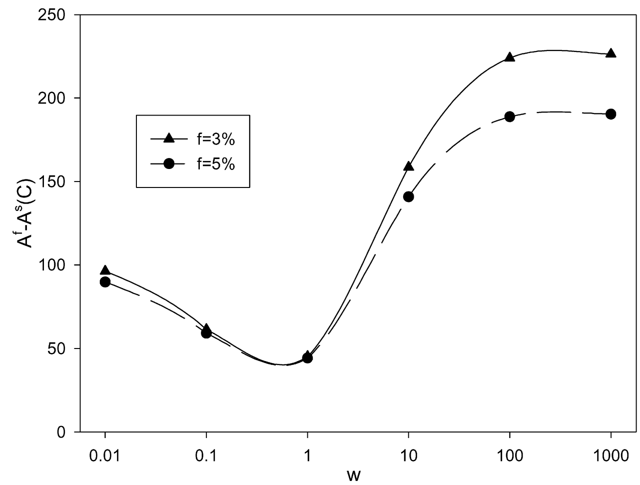

Fig. 3. Variation of $A^f - A^s$ as a function of SMA inclusion's aspect ratio.

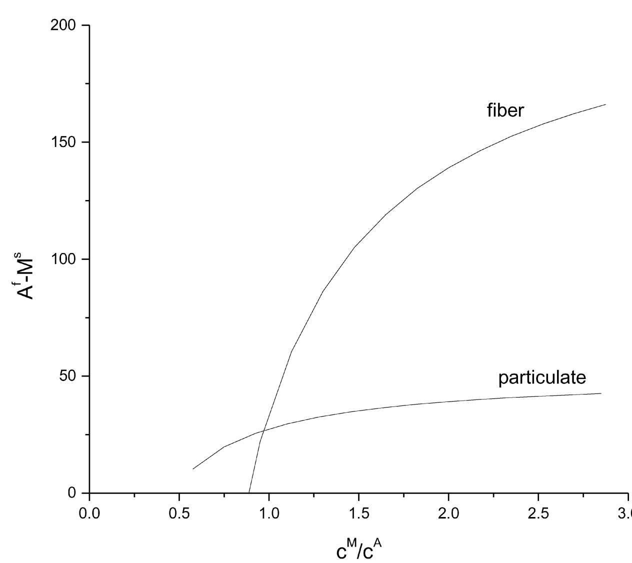

Fig. 4. Variation of $A^f - M^s$ as a function of SMA material characteristic (c^M/c^A) ($f = 3.5\%$).

the composites with the SMA long fibers. To minimize the linear shrinkage, c^{M} should be close to or smaller than c^{A} (we will come back on this point in Section 5). For the considered composite as shown in Fig. 2, a full thermal cycle cannot lead to a complete recover of the martensitic phase for the SMA inclusions.

For the composite with the aligned SMA fibers the transverse thermal expansion of the composite is larger than that of the pure matrix as shown in Fig. 5 owing to the transformation of the SMA fibers. This is due to the additional transverse strain contributed by the SMA inclusions. In order to minimize at the same time the transverse thermal expansion of the composite, SMA fibers must also be placed along the transverse direction, as shown in Fig. 1, and this will be analyzed in the following Section 4.2.

4.2. Composite with two populations of the shape memory alloys inclusions

We will consider the following case: $f^0 = f^{90}$, $\mathbf{S}^0 = \mathbf{S}^{90}$. This means that the same shape and amount of inclusions are placed in two perpendicular directions. From the symmetric consideration, we have $\underline{\underline{\epsilon}}_{\mathrm{T}}^{90} = \mathbf{U} : \underline{\underline{\epsilon}}_{\mathrm{T}}^{0}$, so Eq. (3) now becomes:

$$\underline{\underline{\sigma}}^0 = (\mathbf{M}_{\mathrm{f}} - \mathbf{M}_{\mathrm{m}})^{-1}[(\mathbf{I} - f^0\mathbf{QT})(\mathbf{Q} - \mathbf{I}) - f^0\mathbf{QT}(\bar{\mathbf{Q}} - \mathbf{I})\mathbf{U}] : [(\underline{\underline{\alpha}}_{\mathrm{f}} - \underline{\underline{\alpha}}_{\mathrm{m}})\Delta T + \underline{\underline{\epsilon}}_{\mathrm{mtr}}^{0} - \gamma\underline{\underline{\Lambda}}] \tag{18}$$

In this situation, the composite as a whole is orthotropic, the normal and shear effect are uncoupled. The components of $\underline{\underline{\sigma}}^0$ are now $\sigma_{22}^0, \sigma_{33}^0$ and σ_{11}^0, and the form of $\underline{\underline{\epsilon}}_{\mathrm{mtr}}^{0}$ remains unchanged as in the aligned case owing to the adopted transformation law (Eq. (5)). With the help of Eq. (18), the effective stress in the SMA inclusions can then be related to the current temperature and the transformation strain. Together with Eqs. (5) and (6) for reverse transformation and Eq. (7) for forward transformation, a set of differential equations similar to Eqs. (10) and (12) can be derived for determining the transformation strain $\epsilon_{\mathrm{mtr33}}^{0}$ as function of the temperature. Once these are completed, the overall thermal expansions of the composite can then be calculated with Eqs. (1) and (2).

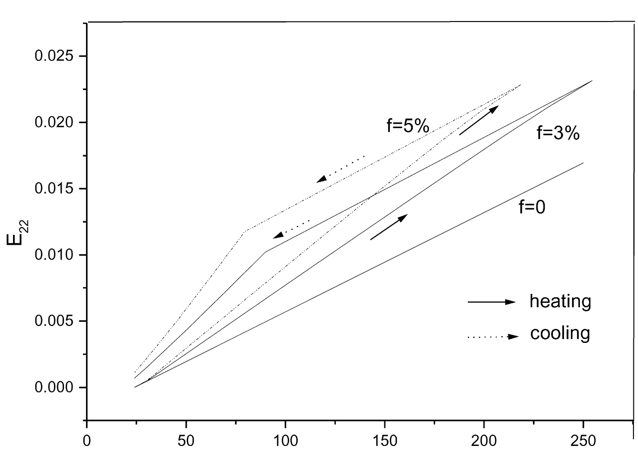

Fig. 5. Transverse thermal expansion of the composite with aligned SMA fibers during a full temperature cycle ($T_0 \to A^{\mathrm{f}} \to T_0$).

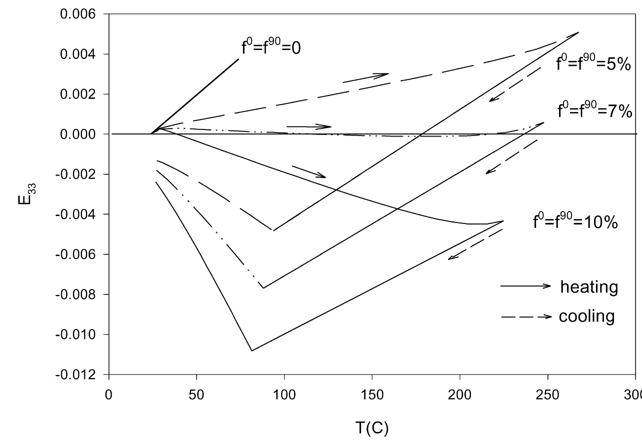

Fig. 6. In-plane thermal expansion for the composite with two populations of SMA fibers perpendicularly oriented during a full temperature cycle ($T_0 \to A^{\mathrm{f}} \to T_0$).

The overall in-plane thermal expansion of the composite is presented in Fig. 6 for the SMA fibers. In the heating process, it is seen that indeed by placing the SMA fibers in horizontal and perpendicular direction, a composite with very small thermal expansion can be obtained (see e.g. Fig. 6 $f^0 = f^{90} = 7\%$ for the SMA fibers). Compared to the aligned case, more SMA inclusions are necessary to compensate the thermal dilatation of the matrix, for example, in the case of the aligned SMA fibers, only 3.5% suffices, however total about 14% (7% for each) is necessary for perpendicularly crossed case. This is due to the interaction occurred between the two populations of the SMA inclusions. During the cooling process, as in the aligned case, the large difference between A^{f} and M^{s} induces a significant linear shrinkage of the composite.

5. Discussions and conclusions

As shown in Section 4, embedding SMA inclusions into a continue matrix can make the composite with large variety of overall thermal expansion behavior. If we now focus on minimizing the overall thermal expansion of the composite, the following two aspects must be considered primarily: (1) $A^{\mathrm{f}} - A^{\mathrm{s}}$ as large as possible to meet the working temperature range. As shown previously SMA fibers are the most suitable for such purpose; (2) $A^{\mathrm{f}} - M^{\mathrm{s}}$ as small as possible to avoid the large linear shrinkage during cooling. Fig. 7 gives the phase transformation diagram for the in situ SMA inclusions in the composite. In order to get small value of $A^{\mathrm{f}} - M^{\mathrm{s}}$, the $M^{0\mathrm{s}}$, $M^{0\mathrm{f}}$, $A^{0\mathrm{s}}$ and $A^{0\mathrm{f}}$ of the SMA inclusions must be as close as possible. In addition the slope of boundary for martensitic domain should be smaller than that of austenite domain (Figs. 4 and 7). The specific

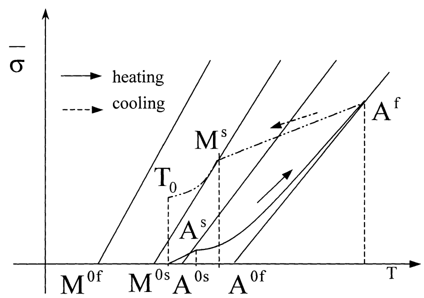

Fig. 7. Sketch of transformation diagram for in situ SMA inclusions in the composite.

material characteristic for the SMA inclusions can be optimized through proper heat treatment. With this idea in mind, we now consider the following SMA material characteristic $c^A = c^M = 4.5$, $M^{0s} = 23$ °C, $M^{0f} = 20$ °C, $A^{0s} = 25$ °C, $A^{0f} = 30$ °C, and the composite is made of two populations of the SMA inclusions with $f^0 = f^{90} = 6.5\%$. Fig. 8 shows the thermal expansion of such optimized composite during a full thermal cycle. Compared with the non-optimized calculation (see Fig. 8, with the previous transformation characteristic) and the pure matrix ($f^0 = f^{90} = 0$), it is clearly seen that by properly designing the microstructure and SMA transformation properties a composite with very small dilatation can be obtained.

To conclude, we proposed a micro-mechanical model to analyze the overall thermal expansion of composites with SMA inclusions. It is shown that embedding of pre-strained SMA inclusions into a continue matrix can make an intelligent composite with specific thermal dilatation behavior. This property depends strongly on the shape, volume concentration, orientation and transformation properties of the SMA inclusions. It is found that SMA fibers are more suitable for large range of temperature variation and that a composite with very small dilatation during a full thermal cycle can be obtained by properly choosing the microstructure and the transformation properties of the SMA inclusions.

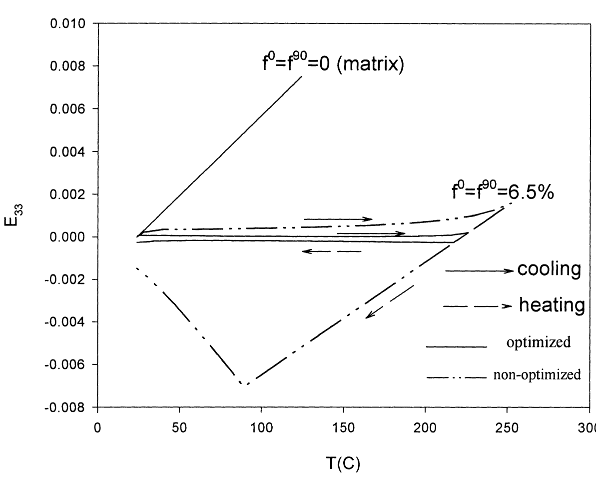

Fig. 8. In-plane thermal expansion for a designed composite with two populations of SMA fibers perpendicularly oriented during a full temperature cycle ($T_0 \rightarrow A^f \rightarrow T_0$).

Finally, it must be emphasized that the analysis presented earlier does not account for the plastic deformation of the matrix, and nor the dependence of the mechanical properties of the matrix on the temperature, and the bond between the matrix and inclusion is assumed perfect. More refined model, which includes these aspects, will be given in the future work.

Acknowledgements

This work is performed during a sabbatical visit of the first author GKH in the Hong Kong University of Science and Technology. The work has been support by the Research Grant Council of The Hong Kong SAR (Project No. HKUST6037/98E) and the National Natural Science Foundation of China (Project No. 19825107). The work of GKH is also supported by National Nature Science Foundation of China through grant 19802003.

Appendix A

A.1. Mori–Tanaka's model

For a composite under a combined temperature change and induced transformation strain, which are simulated by a total eigenstrain $\underline{\underline{\epsilon}}_T$, the average stress and strain in the matrix is related by

$$\underline{\underline{\sigma}}_m = \mathbf{L}_m \underline{\underline{\epsilon}}_m \tag{A1}$$

in the local coordinate with x_3 being the symmetric axis of the inclusion, the stress and strain for the inclusions

$$\underline{\underline{\sigma}}' = \mathbf{L}_f(\underline{\underline{\epsilon}}'_m + \underline{\underline{\epsilon}}'_p - \underline{\underline{\epsilon}}'_T) = \mathbf{L}_m(\underline{\underline{\epsilon}}'_m + \underline{\underline{\epsilon}}'_p - \underline{\underline{\epsilon}}'_T - \underline{\underline{\epsilon}}'_*) \tag{A2}$$

and the Eshelby relation applies

$$\underline{\underline{\epsilon}}'_p = \mathbf{S}'(\underline{\underline{\epsilon}}'_T + \underline{\underline{\epsilon}}'_*) \tag{A3}$$

where the primed quantities mean the expressions in the local coordinate, $\mathbf{S}'$ is the Eshelby tensor in the local coordinate ($\mathbf{S}^0$ for the perpendicularly oriented inclusions and $\mathbf{S}^{90}$ for the horizontally oriented inclusions). Eqs. (A2) and (A3) apply for both perpendicularly and horizontally oriented inclusions.

Since there is no external stress, the volume average (or orientational average) of the local stress over a representative volume element is zero. In the global coordinate, this leads to

$$\underline{\underline{\epsilon}}_m + f^0(\mathbf{S}^0 - \mathbf{I})\left(\underline{\underline{\epsilon}}^0_T + \underline{\underline{\epsilon}}^0_*\right) + f^{90}\mathbf{U}^{-1}(\mathbf{S}^{90} - I)\mathbf{U}\left(\underline{\underline{\epsilon}}^{90}_T + \underline{\underline{\epsilon}}^{90}_*\right) = 0 \tag{A4}$$

Eqs. (A1)–(A4) allow one to determine $\underline{\underline{\epsilon}}_m$, $\underline{\underline{\epsilon}}^0_*$, $\underline{\underline{\epsilon}}^0_p$ and $\underline{\underline{\epsilon}}^{90}_p$, $\underline{\underline{\epsilon}}^{90}_*$ as function of $\underline{\underline{\epsilon}}^0_T$, $\underline{\underline{\epsilon}}^{90}_T$, so are the stresses in the inclusions (Eqs. (3) and (4)). The total strain of the composite is written as $\underline{\underline{E}} = \underline{\underline{\alpha}}_m \Delta T + f^0(\underline{\underline{\epsilon}}^0_T + \underline{\underline{\epsilon}}^0_*) + f^{90}(\underline{\underline{\epsilon}}^{90}_T + \underline{\underline{\epsilon}}^{90}_*)$, and this leads to Eqs. (1) and (2) in Section 2.

A.2. The transformation matrix $\boldsymbol{U}$

$$U = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \cos^2\theta & \sin^2\theta & -2\sin\theta\cos\theta & 0 & 0 \\ 0 & \sin^2\theta & \cos^2\theta & 2\sin\theta\cos\theta & 0 & 0 \\ 0 & \sin\theta\cos\theta & -\sin\theta\cos\theta & \cos^2\theta - \sin^2\theta & 0 & 0 \\ 0 & 0 & 0 & 0 & \cos\theta & \sin\theta \\ 0 & 0 & 0 & 0 & -\sin\theta & \cos\theta \end{bmatrix}$$

$\theta = 90°$ for the considered composite.

A.3. The expressions of the constants h_1, h_2 *and* g_1, g_2, g_3 *and* g_4

$$h_1 = -6b\mu_m\mu_f(1-f)(2s_{1111} + s_{1133} - 2s_{1212} - 2s_{3311} - s_{3333})/[F(\mu_m - \mu_f)]$$

$$h_2 = 3b(1-f)\mu_m\mu_f[t_1 - 3a(1-f)t_3]/[F(\mu_m - \mu_f)]$$

$$g_1 = 3[1 + b(1-f)(1 + s_{1133} - s_{3333})]/F$$

$$g_2 = -3[1 + a(1-f)(1 - 2s_{1133} - s_{3333})]/(2F)$$

$$g_3 = 3[1 + b(1-f)(1 - 2s_{1111} + 2s_{1212} + 2s_{3311})]/F$$

$$g_4 = 3[1 + a(1-f)(1 - 2s_{1111} + 2s_{1212} - s_{3311})]/F$$

where $F = [3 - b(1-f)t_1 - a(1-f)t_2 + 3ab(1-f)^2 t_3]$

and

$$t_1 = -3 + 2s_{1111} - 2s_{1133} - 2s_{1212} - 2s_{3311} + 2s_{3333}$$

$$t_2 = -3 + 4s_{1111} + 2s_{1133} - 4s_{1212} + 2s_{3311} - s_{3333}$$

$$t_3 = 1 - s_{3333} - 2s_{1133}s_{3311} + 2(s_{1111} - s_{1212})(-1 + s_{3333})$$

and s_{ijkl} is the components of Eshelby tensor with x_3 as the symmetric axis.

References

[1] Sun QP, Hwang KC. Micromechanics modeling for the constitutive behavior of polycrystalline shape memory alloys: I. Derivation of general relations; II. Study of the individual phenomena. J Mech Phys Solids 1993;41:1–33.

[2] Lu ZK, Weng GJ. Martensitic transformation and stress-strain relations of shape-memory alloys. J Mech Phys Solids 1997;45:1905–28.

[3] Huang MS, Brinson LC. Multivariant model for single crystal SMA behavior. J Mech Phys Solids 1998;46:1379–409.

[4] Liang C, Rogers CA. A multi-dimension constitutive model for shape memory alloys. J Engng Math 1992;26:429–43.

[5] Sato Y, Tanaka K. Estimation of energy dissipation in alloys due to stress-induced martensitic transformation. Res Mech 1988;23:381–93.

[6] Boyd JG, Lagoudas DC. A thermodynamical constitutive model for shape memory materials, Part I: the monolithic shape memory alloy. Int J Plasticity 1996;12:805–42.

[7] Birman V. Review of mechanics of shape memory alloy structures. Appl Mech Rev 1997;50:629–45.

[8] Lagoudas DC, Boyd JG, Bo Z. Micromechanics of active composite with SMA fibers. J Engng Mater Technol 1994;116:337–47.

[9] Cherkaoui M, Sun QP, Song GQ. Micromechanics modeling of composite with ductile matrix and shape memory alloy reinforcement. Int J Solids Struct 2000;37:1577–94.

[10] Song GQ, Sun QP, Cherkaoui M. Role of microstructures in the thermomechanical behavior of SMA composites. J Engng Mater Technol 1999;121:86–92.

[11] Qiu YP, Weng GJ. On the application of Mori–Tanaka's theory involving transversely isotropic spheroidal inclusions. Int J Engng Sci 1991;28:1121–38.

[12] Mura T. Micromechanics of defects in solids. Dordrecht, Boston: Martinus Nijhoff Publishers, 1987.