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Experimental study for metamaterials based on dielectric resonators and wire frame

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Abstract

A double-negative (DNG) material based on dielectric particles is experimentally analyzed. The DNG material is assembled using ceramic particles with high dielectric constant, and a conducting wire frame. The ceramic particles contribute to the effective negative permeability thanks to Mie resonance, while the conducting wire frame provides the effective negative permittivity. The measured transmittances are in good agreement with the theoretical prediction. The influence of the particle shape and distribution on the effective negative permeability is also examined experimentally. We show that random distribution of the particles hinders collective response.

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Keywords: Double-negative material; Metamaterial; Negative permeability; Mie resonance

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1. Introduction

Since the first double-negative (DNG) material [1] reported by Shelby et al. [2,3], much progress has been made for searching new DNG materials and their potential applications [4–7]. DNG materials typically rely on the microstructure suggested by Pendry et al. [8], being composed of split-ring resonators (SRRs) and conducting wires. The SRRs are used to generate magnetic resonance and to provide negative permeability. However, there are inherent difficulties to fabricate three-dimensional DNG materials with this method. In addition, the SRRs are difficult to be made electrically small at optical frequencies.

In order to overcome these problems, DNG materials based on resonant dielectric particles seem to be promising. The basic principle was reported by Lewin [9] in 1947, and recently used for metamaterial design. Holloway et al. [10,11] proposed an isotropic DNG material with magnetodielectric particles embedded in a dielectric matrix, with the particles supposed to have high values of both the permittivity and permeability. Vendik et al. [12,13] and Jylhä et al. [14] proposed a DNG material based on two sets of particles with different sizes. By adjusting the sizes of the two populations of the particles, they show that the composite can have electric and magnetic resonances over a common frequency range. Wheeler et al. [15] suggested coated particles to trigger simultaneously the electric and magnetic resonances. In that model, two additional variables (i.e. diameter and dielectric constant of the coating material) are introduced, so that effective negative permittivity and permeability can be easily realized over a common

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frequency. O'Brien and Pendry [16] theoretically analyzed a DNG material with dielectric cylinders, and the results has been supported experimentally by Peng et al. [17].

Most of the works on DNG materials with dielectric particles concentrate on theory, while there are not so many experiments reported (see e.g. works by Peng et al. [17], Baker-Jarvis et al. [18] and Holloway et al. [19]). In this paper, we will further analyze DNG materials based on dielectric particles and also study the influence of particle shape and their arrangement. The basic theory for predicting the effective property and simulation results is given in Section 2. The experimental characterization and its comparison with the theoretical prediction is presented in Section 3.

2. Theoretical analysis

2.1. Analytical modeling of effective negative permeability

When electromagnetic waves enter a medium containing electrically small obstacles, the overall scattering effect of the medium can be characterized by the effective permittivity and permeability. For a single isolated sphere of radius r_0 with a relative permittivity $\varepsilon_r = n^2$ (n is the refractive index of the sphere), under a plane wave illumination, the scattered field can be decomposed into multipole terms [9,15,20,21]. The mth order electric scattering coefficients a_m can be expressed by:

$$a_\mathrm{m} = \frac{n\psi_\mathrm{m}(nx)\psi'_\mathrm{m}(x) - \psi_\mathrm{m}(x)\psi'_\mathrm{m}(nx)}{n\psi_\mathrm{m}(nx)\xi'_\mathrm{m}(x) - \xi_\mathrm{m}(x)\psi'_\mathrm{m}(nx)}, \tag{1}$$

and the mth order magnetic scattering coefficients b_m can be expressed by:

$$b_\mathrm{m} = \frac{\psi_\mathrm{m}(nx)\psi'_\mathrm{m}(x) - n\psi_\mathrm{m}(x)\psi'_\mathrm{m}(nx)}{\psi_\mathrm{m}(nx)\xi'_\mathrm{m}(x) - n\xi_\mathrm{m}(x)\psi'_\mathrm{m}(nx)}. \tag{2}$$

where $x = k_0 r_0$, $k_0 = \omega/c$ is the wave vector in vacuum, and ψ_m and ξ_m are the Riccati–Bessel functions, which are related to spherical Bessel functions by $\psi_\mathrm{m}(z) = z_{\mathrm{jm}}(z)$ and $\xi_\mathrm{m}(z) = zh^{(1)}_\mathrm{m}(z)$. The scattered magnetic dipole field is the same compared with the magnetic dipole radiation if an effective magnetic dipole polarizability is introduced [9,15,20,21]:

$$\alpha_1 = \frac{6\pi i b_1}{k_0^3}. \tag{3}$$

According to Clausius–Mossotti equation [22], in the long wavelength (or electrically small) limit, the effective permeability μ_eff of the medium containing small particles can be written in terms of the magnetic dipole polarizability:

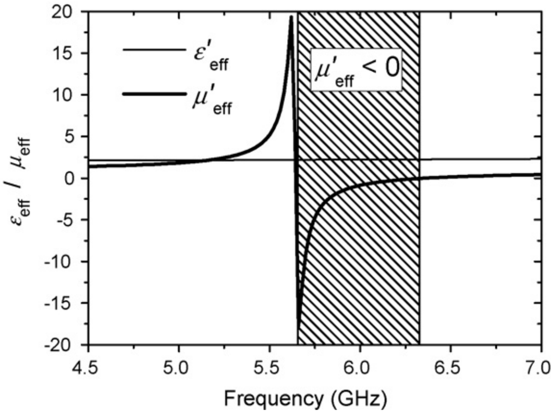

Fig. 1. Calculated effective permeability and permittivity of a composite build up with dielectric particles embedded in a polymer foam.

$$\mu_\mathrm{eff} = 1 + \frac{N\alpha_1}{1 - N\alpha_1/3}. \tag{4}$$

Here N is the number of particles in a unit volume, and it is related to the volume fraction f of the particle by $f = 4\pi N r_0^3/3$.

With help of Eq. (3), the effective permeability is further written as

$$\mu_\mathrm{eff} = \frac{2(k_0 r_0)^3 + 6ifb_1}{2(k_0 r_0)^3 - 3ifb_1}. \tag{5}$$

Following the same process, the effective permittivity is expressed as

$$\varepsilon_\mathrm{eff} = \frac{2(k_0 r_0)^3 + 6ifa_1}{2(k_0 r_0)^3 - 3ifa_1}. \tag{6}$$

We consider a composite with dielectric particles periodically embedded in a polymer foam. Effective permittivity and permeability of the composite are estimated by Eqs. (5) and (6), with the results shown in Fig. 1. In the computation, the particle is assumed to be sphere of radius $r_0 = 2.7$ mm and its dielectric constant is $\varepsilon_\mathrm{r} = 88.3$, the dielectric constant of the polymer foam is $\varepsilon_0 = 1.05$. The lattice constant and the filling fraction are respectively $a = 6.7$ mm and $f = 27.4\%$. The effective permittivity of the composite is estimated to be around $\varepsilon_\mathrm{eff} = 2.3$ over all the calculated frequency range (4.5–7 GHz), while the effective permeability is found to be negative in a band extended from $\omega_\mathrm{low} = 5.64$ GHz to $\omega_\mathrm{up} = 6.34$ GHz. Here ω_low is the resonant frequency at which the denominator of Eq. (2) equals to zero, and ω_up is the frequency at which the effective permeability passes through zero, i.e. $\mu_\mathrm{eff} = 0$. The electric resonance takes place at a higher frequency, which is not shown

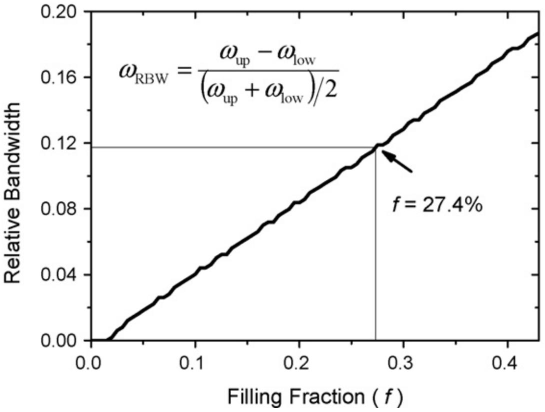

Fig. 2. Variation of the relative bandwidth for the negative permeability as a function of the filling fraction of the dielectric particles.

here. At the higher-frequency edge of the negative permeability band, the ratio of the operating wavelength to the lattice constant is about $\lambda/a = 8$. Generally, this ratio is proportional to the relative permittivity of the dielectric particle.

The absolute range of the band with negative permeability ($\mu_{\mathrm{eff}} < 0$) in Fig. 1 is about $\omega_{\mathrm{up}} - \omega_{\mathrm{low}} = 0.7$ GHz. This bandwidth decreases with either increasing the dielectric constant or reducing the radius of the particle [15]. However, the relative bandwidth (RBW) defined by $\omega_{\mathrm{RBW}} = 2(\omega_{\mathrm{up}} - \omega_{\mathrm{low}})/(\omega_{\mathrm{up}} + \omega_{\mathrm{low}})$ is not significantly influenced by the dielectric constant or the radius of the particle. Fig. 2 shows the RBW as function of the filling fraction of the particle, it is seen that by increasing the filling fraction, the RBW is almost linearly increased. An approximate relation of $\omega_{\mathrm{RBW}} = f/2$ is found, which is consistent with other magnetic resonators [23].

2.2. *Design for double negative materials*

From the above analysis, the effective permittivity and permeability of the composite can be negative over certain frequency range, however it is difficult to tune them negative simultaneously with one population of particle [15]. To remedy this, coated particles [15,24] or particles with different sizes [12–14,24] have been proposed. However, these approaches may still be challenging with current fabrication technology. So in this paper, we propose a DNG material built up with ceramic spheres and a conducting wire frame, as illustrated in Fig. 3. The ceramic spheres are periodically distributed in a conducting wire frame, which simulates an artificial plasma medium with a negative permittivity, and the ceramic spheres drive magnetic resonance in this background. This approach is similar to the case where the SRR elements are filled into a “small-size waveguide” to realize a DNG material [25,26].

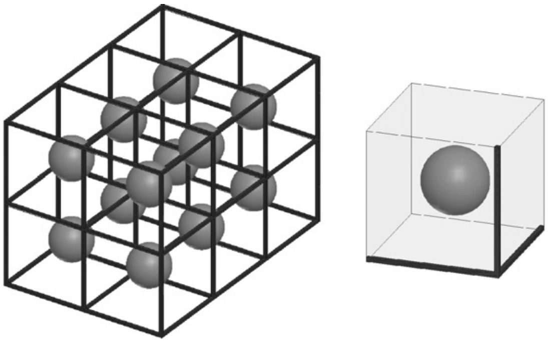

Fig. 3. Scheme of a 3D DNG metamaterial with dielectric particles and a conducting wire frame.

The transmission of a standard waveguide WR-159 loaded with ceramic spheres, metal wires and the composite (conducting wires and ceramic spheres, as shown in Fig. 3) are simulated by Ansoft HFSS, the results are shown in Fig. 4. In the computation, the radius of the wire is 0.5 mm and the lattice constant is 6.7 mm. The dielectric constant of the ceramic sphere is the same as that used in Section 2.1.

Due to the negative permeability provided by the ceramic spheres, the transmission of the waveguide is very low (about −80 dB) within a frequency band from 5.64 to 6.34 GHz. In addition, due to the negative permittivity provided by the metal wires (without ceramic spheres), the transmission is also lower than −100 dB throughout the examined frequency range. However, the transmission of the waveguide loaded with the composite turns up to −10 dB within the same frequency band from 5.64 to 6.34 GHz, while the transmissions in other frequency ranges remain very low. The electromagnetic wave propagates forward in a traditional material. So

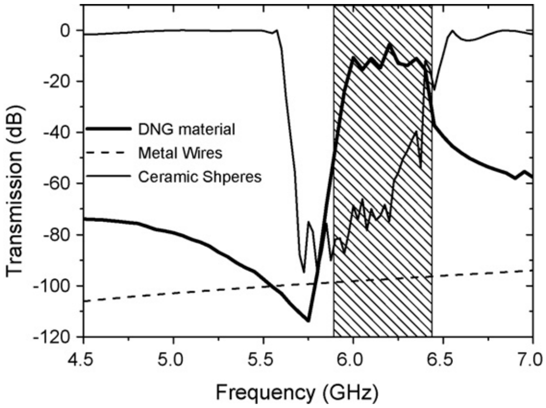

Fig. 4. Simulated transmission coefficients of the waveguide loaded with metal wires, ceramic spheres or the combined metamaterial.

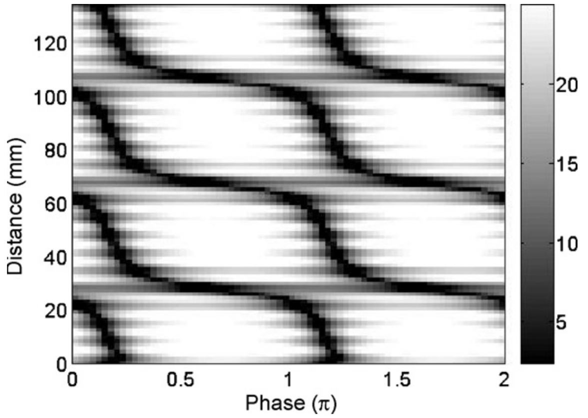

Fig. 5. Simulated electric field pattern along the central line of the DNG material at 6.20 GHz.

if the electric field is plotted on the space-time diagram, the slope of the constant phase (or constant electric field) will be positive. However, for a DNG material, the slope of the constant phase will be negative [27]. Fig. 5 shows the electric field through the central line of the material, which is extracted from simulation result at the frequency of 6.20 GHz. The thick black lines representing the constant phase of the electric field have negative slopes, this confirms the backward wave propagating property of the composite (conducting wires and ceramic spheres). So the above design offers a DNG material in the frequency range 5.64–6.34 GHz. The electric field distribution also shows unmatched impedance inside the material, which leads to multi-reflection and standing waves. In the following, we will examine experimentally the transmission properties for the proposed model.

3. Experimental analysis

3.1. Composite without conducting wire frame

In experiment, the layers of polymer foams containing periodically arranged ceramic spheres are placed inside of a waveguide. The ceramic sphere is made of $BaCO_3$–TiO_2, with a dielectric constant of $\varepsilon_r = 88.3$, and its radius is $r_0 = 2.7 \pm 0.05$ mm. To fix the spheres, the layers of polymer foams (40.4 mm × 20.2 mm × 6.7 mm) with dielectric constant $\varepsilon_0 \leq 1.05$ are used. The particles are placed periodically into the layer of the polymer foam with a lattice constant $a = 6.7$ mm. A sample with five such layers is put into a standard waveguide WR-159, the transmission property of the waveguide is measured by the Agilent network analyzer E8362B. Different samples are also fabricated to examine the influence of the shape and arrangement of the particles. Fig. 6 shows the samples with the spherical particles periodically distributed, the spherical particles randomly arranged and cylindrical particles periodically arranged, respectively. For the cylindrical particles, height is set to be equal to the diameter.

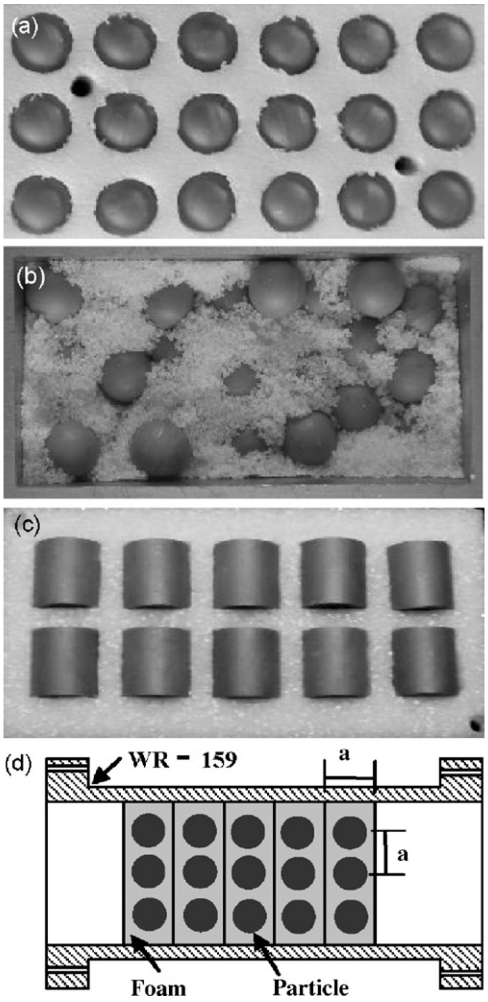

Fig. 6. Photographs of the samples: (a) ceramic spheres periodically embedded in the polymer foam, (b) ceramic spheres randomly embedded in the polymer foam, (c) ceramic cylinders periodically embedded in the polymer foam, (d) sketch of a sample placed in the WR-159 waveguide.

Fig. 7 shows the measured transmission coefficients for the samples with periodically and randomly arranged particles. The results show that the stopbands appear in both the periodic and the random cases. For the sample with the periodically distributed particles, the measured transmittance is in good agreement with the numerical prediction showed in Fig. 4. In the stopband, the transmission of the sample with the randomly distributed particles is much higher than that with the

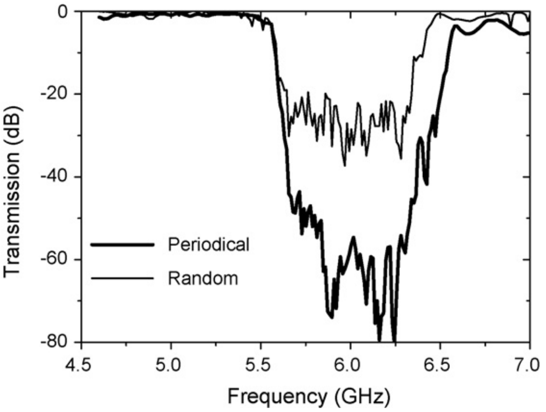

Fig. 7. Measured transmission coefficients of the composites containing periodically or randomly distributed spheres.

periodically arranged particles. According to Gorkunov et al. [28], disorder of the particles leads to damping of the resonance, thus, more power is transmitted through the sample with the randomly distributed particles. For this reason, the periodically arranged particles are suggested in the experiment to realize a DNG material.

As long as the particles are electrically small, the bulk electromagnetic response of a medium containing such particles can be characterized by the effective parameters. To examine the influence of the particle shape, the sample with cylindrical particles has also been fabricated, the measured transmission is shown in Fig. 8. Roughly, the Mie resonance is independent of particle shape. To compare the result with that of the spherical particle, a transformation from a cylinder to an equivalent sphere is made by keeping their volumes constant. Thus, the equivalent radius of the cylindrical particle can be defined as $r = \sqrt[1/3]{2/3}\,r_0$ (the height of the cylinders is set to be equal to the diameter). Therefore, the effective permeability of the sample built up with cylindrical particles can be estimated from the model for the spherical particles by Eq. (6). The calculated effective permeability is illustrated in Fig. 8 by the solid line. The frequency range of negative permeability ($\mu_{\mathrm{eff}} < 0$) is in good agreement with the measured stopband. However, the measured transmission curve in the ascending part (5.0–5.4 GHz) is not as smooth as that with the spherical particle. This is likely to be caused by a higher-order scattering at the edges of the cylindrical particles. The influence of cylinders alignment is also examined the results show that no significant difference is observed for the cylinders arranged following in the vertical, horizontal and transverse directions. Even a composite with cubic particles would show the similar transmission pattern if the filling fraction is kept unchanged [29].

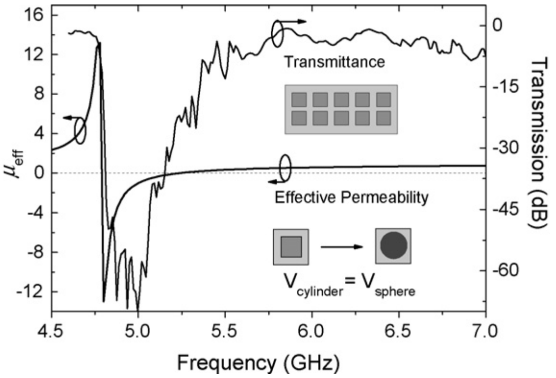

Fig. 8. Measured transmittance and calculated effective permeability of the composite containing periodically arranged cylinders.

3.2. Composite with conducting wire frame

To measure the transmission characteristics of the proposed DNG material, the waveguide (WR-159) is modified by drilling periodic holes on both the upper and lower broadwalls. The diameter of the holes is 1.0 mm, and the lattice constant is 6.7 mm. Copper wires with diameter 1.0 mm are vertically inserted through the holes across the waveguide (as shown by Fig. 9a). Since the wires are parallel to the electric field of the TE_{10}

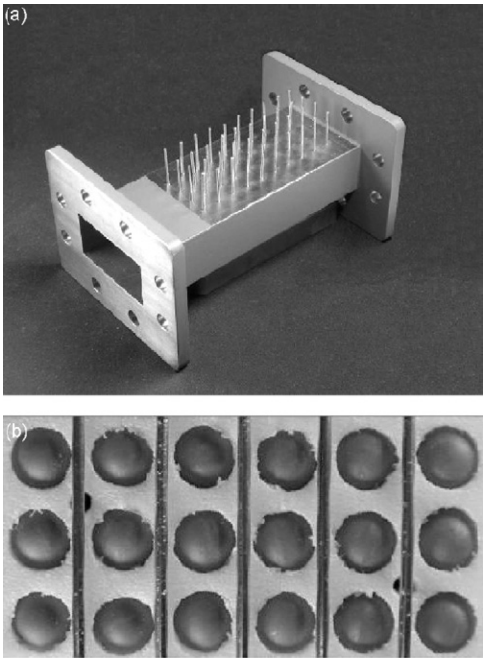

Fig. 9. Photographs of the designed waveguide with metal wires inserted (a) and the sample of the proposed DNG material (b).

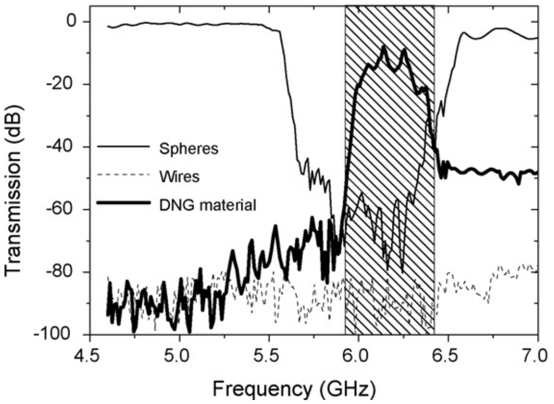

Fig. 10. Measured transmission coefficients of the waveguide loaded by metal wires, ceramic spheres and the proposed DNG material, respectively.

mode in the rectangular waveguide, the special WR-159 waveguide simulates the background with a negative permittivity as discussed in Section 2.2.

The polymer foam layers containing the ceramic spheres are placed inside the special waveguide, the section plane of the composite is illustrated in Fig. 9b. The transmission coefficient of the composite-loaded waveguide is measured and shown by the thick solid line in Fig. 10. For comparison, the transmittances of the waveguide loaded with only ceramic spheres and the special waveguide (loaded with wires) are also shown. We note that the measured transmission properties of the composite copper wires and ceramic spheres are in good agreement with the numerical predictions shown in Fig. 4. The transmission spectrum of the composite exhibits a passband (shown with shadow in Fig. 4) which is also in good agreement with the negative permeability frequency range predicted by Mie theory (see Fig. 1). The simulated electric field on the space-time diagram indicates clearly negative slopes (see Fig. 5), which indicates backward wave propagation.

The passband of the DNG material has a peak transmission of −10 dB in Fig. 10. The loss is mainly due to the unmatched impedance of the DNG material to vacuum. The estimated plasma frequency of the metal wire is around 20 GHz [30], and the absolute value of the effective permittivity (provided by the wires) of the DNG material reaches 15. So the reflection is very large, leading to the observed drop in the transmission. Further reduction of reflection in this DNG material would require thinner wires or greater separation of the wires, so that the plasma frequency of the wires would be lower. In addition, dissipation in ceramic spheres dampens the resonant behavior of the composite, and contributes to the overall losses in the DNG material. The dielectric loss tangent of the ceramic sphere should be selected less than a threshold value [10], otherwise no resonance would occur. The small peaks appeared in the passband in Fig. 10 are attributed to the finite thickness of the DNG material along the wave propagation direction [31]. With the increase in the number of the layers, the transmission curve is expected to be smoother.

4. Conclusions

In this paper, we have experimentally analyzed the transmission coefficient of a DNG material build up with ceramic particles and a conducting wire frame. In a certain frequency range, the ceramic particles with high dielectric constant provide negative permeability due to the Mie resonance. The conducting wire frame with orthogonally arranged copper wires leads to negative effective permittivity. By combining these two systems, a DNG material is assembled and characterized. The transmission properties of the DNG material in a waveguide are measured, and the experimental results compare favorably with the numerical prediction. The influence of shape and arrangement of the ceramic particle on the effective properties is also studied experimentally. We observed that the shape of the particles has a predictable effect on the resonance frequency of the permeability, and that the disorder in particles arrangement dampens the resonance.

Acknowledgements

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