Experimental study on electromagnetic wave transparency for coated metallic cylinders

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In this work, coated metallic cylinders transparent to electromagnetic waves are designed based on the neutral inclusion method; they are then verified experimentally by measuring the reflection coefficient and total scattering fields. The dynamic effective permittivity of a composite made of coated metallic cylinders is first derived by setting the effective permittivity of the composite equals to that of air; the necessary dielectric constant of the coating layer as function of the geometry of the coated cylinder is obtained. We choose the mixture of carbon nanotube (CNT) and rubber for the coating material, its dielectric constant can be tuned by adjusting the content of CNT to reach the value necessary for realizing the transparency. The reflection coefficients and the total scattering electric fields are measured for the carefully designed coated and bare copper cylinders, respectively, significant reductions of the reflection coefficient and total scattering fields are observed for the coated cylinders, which confirms the proposed design method. © *2009 American Institute of Physics*. [DOI: 10.1063/1.3132864]

I. INTRODUCTION

Transparency here means that an object placed in an electromagnetic (EM) wave field as if it were not there, it neither scatters nor absorbs the EM wave. Basically, there are two different methods for realizing the so-called transparency, one is based on transformation optics, leading to cloak devices,[1–3] the other is relied on the method of scattering cancellation[4,5] or neutral inclusion concept.[6] The transparency design based on transformation optics is built on geometry-material equivalence due to the invariant property of Maxwell equation under a space transformation. The propagation of an EM wave can be guided to go around a region where anything inside of this region is transparent to the EM wave. After the idea put forward by Pendry *et al.*[2] and Leonhardt,[1] this method has been applied by many authors to establish the function of cloak devices and the material parameter necessary for their realization.[7–10] The design based on the transformation optics usually needs an anisotropic cloak layer with a spatial variation of the material property. The method of the scattering cancellation proposed by Alù and Engheta[4,5] is to directly reduce the total scattering cross section of an object by including plasmonic covers. Under the transparency condition, the object with the cover can be considered as a neutral inclusion, which will not scatter the incident wave.[6] This leads naturally to the neutral inclusion concept,[11] i.e., if the effective permittivity and permeability of the object is the same as that of the surrounding material, the object will be transparent to an EM wave. The advantage of scattering cancellation method is that the coating layer can be made of an isotropic material, which is easily realized in practice. Although there are many theoretical predictions on the transparency with help of scattering cancellation or neutral inclusion method,[4–6,12–16] the corresponding experimental verifications are still necessary. In this paper, we are interested in the neutral inclusion method, especially in experimental validation of the predicted transparency phenomenon. The paper will be arranged as follows: the neutral inclusion method is first applied to design a transparent coated metallic cylinder; the necessary dielectric constant of the coating material is derived as function of geometry of the coated cylinder. This will be explained in Sec. II. The fabrication of the coating material and the experimental validation for the designed transparency will be presented in Sec. III, and followed by conclusions.

II. DESIGN OF TRANSPARENCY FOR A COATED METALLIC CYLINDER

We will consider the following problem: an infinitely long coated metallic cylinder is under an illumination of plane EM wave, the symmetric axis of the cylinder is parallel to the **E** field [Fig. 1(a)]. We will seek the material for the coating layer in order to make the coated cylinder as a whole transparent to the EM wave. Here the cylinder is assumed to be metallic and the coating layer is an isotropic and homogeneous dielectric material. According to the neutral inclusion concept, if the effective permittivity of a composite

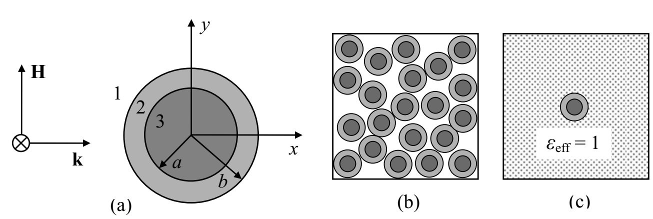

FIG. 1. (a) Scattering of a plane wave by a metallic cylinder coated with a dielectric layer. (b) A composite with randomly distributed coated cylinders. (c) Effective medium model and neutral inclusion concept.

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made of coated cylinders is tuned to be equal to that of air, the coated cylinder will be transparent for the EM wave in air, at least, in quasistatic approximation. So the evaluation of the effective permittivity of the composite made of coated cylinders is a crucial point for the transparency design. In the following, we will apply the homogenization method to evaluate the effective permittivity and in turn to design the material parameters of the coating layer in order to realize the transparency.

We consider a composite assembled by the coated cylinders [see Fig. 1(b)], the coated cylinders (eventually with varying sizes) are placed randomly to the space. For a long metallic cylinder with a dielectric coating, under a plane wave illumination, the incident and the scattered electric fields in the region (1) [see Fig. 1(a)] can be expressed as:[17,18]

$$E_z^{\text{inc}} = E_1^i = E_0 e^{ik_1\rho\cos\theta} = E_0 \sum_{-\infty}^{+\infty} (i)^n J_n(k_1\rho) e^{in\theta}, \quad (1)$$

$$E_1^{\text{sc}} = E_0 \sum_{-\infty}^{+\infty} (i)^n A_n H_n^{(1)}(k_1\rho) e^{in\theta}. \quad (2)$$

The electric field in the region (2) can be written in the following form:

$$E_2 = E_0 \sum_{-\infty}^{+\infty} (i)^n [B_n H_n^{(2)}(k_2\rho) + C_n H_n^{(1)}(k_2\rho)] e^{in\theta}, \quad (3)$$

where A_n, B_n, and C_n are the scattering coefficients and determined from proper boundary conditions. k_1 and k_2 are the wave numbers for the matrix and coating, respectively. J_n and N_n are Bessel functions of the first and second kinds, while $H_n^{(1)} = J_n + iN_n$ and $H_n^{(2)} = J_n - iN_n$. Since the cylinder is metallic, there is no electric field in the region (3). The corresponding magnetic fields in each region can be obtained from the Maxwell's equation $\nabla \times E = i\omega\mu H$, once the electric fields are known.

The scattering coefficients A_n, B_n, and C_n in Eqs. (1)–(3) are determined with the help of the appropriate boundary conditions at $\rho = a$ and $\rho = b$, where a is the inner radius of the cylinder and b is the radius of the coating. This leads to[17]

$$A_n = \frac{k_2 J_n(k_1 b) G_n - k_1 J_n'(k_1 b) M_n}{-k_2 H_n^{(1)}(k_1 b) G_n + k_1 H_n'^{(1)}(k_1 b) M_n},, \quad (4)$$

where a prime means a derivative with respect to the argument and

$$G_n = H_n'^{(1)}(k_2 b) H_n^{(2)}(k_2 a) - H_n'^{(2)}(k_2 b) H_n^{(1)}(k_2 a),$$

$$M_n = H_n^{(1)}(k_2 b) H_n^{(2)}(k_2 a) - H_n^{(2)}(k_2 b) H_n^{(1)}(k_2 a).$$

The magnetic field has a jump at the boundary $\rho = a$, therefore an electric current is induced on the surface of the metallic cylinder. The electric current density can be estimated by:

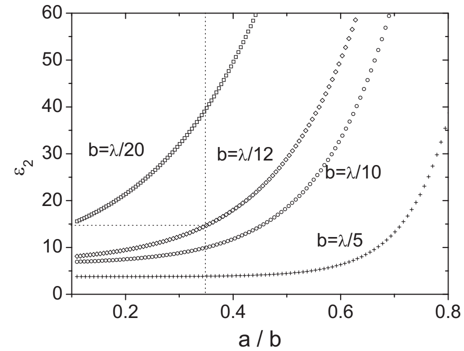

FIG. 2. The necessary permittivity of the coating material as function of the ratio of core-coating radii for different wavelengths.

$$j_z = -H_\theta\big|_{\rho=a} = -\frac{1}{i\omega\mu_2}\left.\frac{\partial E_z}{\partial\rho}\right|_{\rho=a} = -\frac{E_0 k_2}{i\omega\mu_2}\sum_{-\infty}^{+\infty}(i)^n [B_n H_n'^{(2)} \times (k_2 a) + C_n H_n'^{(1)}(k_2 a)] e^{in\theta}. \quad (5)$$

This surface current contributes to the electric displacement of the composite; the spatial average of the current over the composite element can be defined as:

$$\langle I_z \rangle = \frac{1}{\pi b^2} \oint_{2\pi} j_z a\, d\theta. \quad (6)$$

The effective conductivity σ_{eff} along the axis of the cylinder can be defined as

$$\langle I_z \rangle = \sigma_{\text{eff}} \langle E_z \rangle \quad (7)$$

and $\langle E_z \rangle$ is the spatial average of the electric field in z direction over the region (2),

$$\langle E_z \rangle = \frac{1}{\pi b^2} \int\int_{\rho=a,b;\theta=0,2\pi} E_2 \rho\, d\rho\, d\theta. \quad (8)$$

Once the effective conductivity of the composite is known, the effective permittivity in z direction can be derived as:

$$\varepsilon_{\text{eff}} = \varepsilon_2 + i\frac{\sigma_{\text{eff}}}{\omega}. \quad (9)$$

According to the neutral inclusion method, when the effective permittivity of the composite is set to be the same as that of air, the coated cylinder will be transparent to the EM wave. This condition will give the relation between the material parameters of the coating layer and the geometry of the coated cylinder necessary for the transparency. Figure 2 shows the dependence of the permittivity of the coating layer as function of the cylinder radius under the transparency condition. For a given wavelength and a fixed thickness of the coating layer, it is found that the necessary permittivity of the coating layer increases with the radius of the metallic cylinder (i.e., the volume fraction of the cylinder). We also see that for a given cylinder radius, the coating layer with a higher permittivity makes the cylinder transparent for a longer wavelength (i.e., a lower frequency).

Now we fix the geometry of the coated cylinder, say $a = 1.0$ mm and $b = 3.0$ mm. Figure 3 shows the variation of

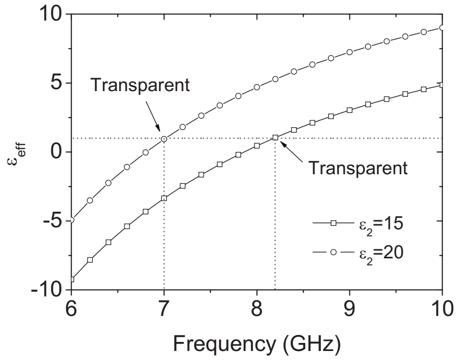

FIG. 3. Calculated effective permittivity of the coated cylinders as function of the incident frequency.

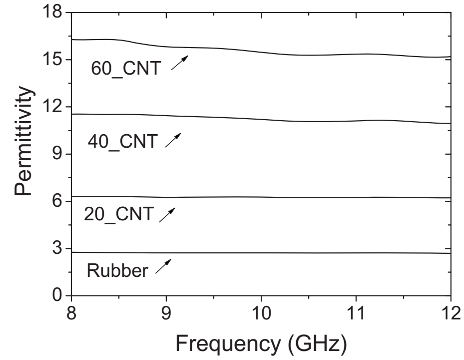

FIG. 5. The measured dielectric constants of the composites made of CNTs and rubber for different mass fractions of the CNT.

effective permittivity of the coated cylinder and is shown as function of the incident frequency for two kinds of coating $\varepsilon_2=20$ and $\varepsilon_2=15$. The neutral inclusion method predicts that the coated cylinders will be transparent to the incident wave at about 7.0 and 8.2 GHz for $\varepsilon_2=20$ and $\varepsilon_2=15$, respectively.

To check these predictions, we examine in the following the total scattering cross section (TCS) of the coated cylinders, which is defined as:

$$\sigma_{\text{total}} = \frac{4}{k_1}\sum_{-\infty}^{\infty} A_n A_n^*, \tag{10}$$

where A_n is given by Eq. (4) and A_n^* means its conjugate.

The total cross section of a metallic cylinder (without coating) increases linearly with the frequency, as shown by the solid line in Fig. 4. When the same cylinders are coated by the materials with dielectric constants of $\varepsilon_2=20$ and $\varepsilon_2=15$, their TCSs tend to minimums at 7.0 and 8.2 GHz, respectively, as expected. In the following, we will examine experimentally the predicted transparency; the geometry of the coated cylinder is chosen to be $a=1.0$ mm and $b=3.0$ mm. So if the dielectric constant of the coating is $\varepsilon=15$, the coated cylinder will be transparent at 8.2 GHz.

III. EXPERIMENTAL STUDY

The transparency of coated cylinders can be demonstrated by measuring its backscattering field or total scattering field. A transparent object will display low backscattering, thus the reflection coefficient of an array of such designed coated cylinders will be very low compared to that of bare cylinders. The other technique to verify the transparency is to scan the induced total scattering field for the coated and bare cylinders placed in an EM wave field. In the following, we will use these two techniques to examine the predicted transparency phenomenon of the coated cylinders.

To make the coating material, we focus on the composite made of carbon nanotube (CNT) and rubber; the dielectric constant of the composite can be tailored by varying the content of the CNT. The mixture of CNT and rubber is processed by spray drying technique; the detailed procedure is given in Ref. 19. The permittivity of the composite (coating layer) at X-band is measured by a vector network analyzer (Agilent E8362B) through standard testing process. Figure 5 shows the dielectric constants of the composite for the different fractions (mass) of nanotubes; it is found that the composite with the ratio of CNT and rubber 60:100 has a permittivity around 15 throughout the examined frequency, which meets our requirement. So we in the following choose the composite with CNT and rubber ratio of 60:100 as the coating material.

The flexible coating material is of a plate shape with 2 mm thickness; it is then tailed to small concentric loops with an inner radius 1.0 mm and an outer radius 3.0 mm. Copper cylinders are inserted through the loops, so the designed coated cylinders are then fabricated. In the experiment, arrays of the coated and bare cylinders are placed into a special waveguide, and the S_{11} parameters of the waveguide are measured. Figures 6(a) and 6(b) show the waveguides loaded by an array of the coated cylinders and the bare cylinders,

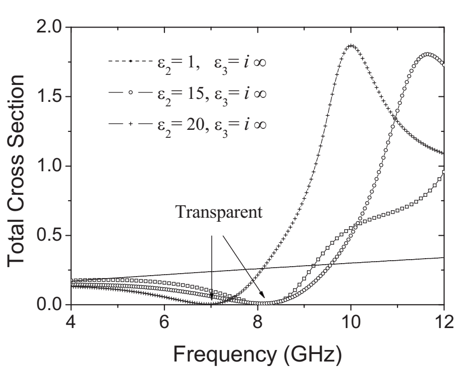

FIG. 4. Calculated total TCS of the coated cylinders as function of the incident frequency for different coating materials.

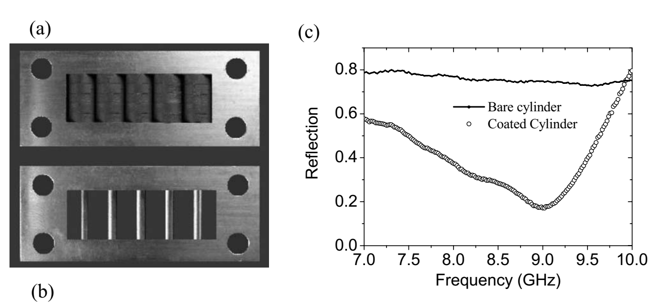

FIG. 6. (a) Waveguide loaded by coated cylinders, (b) waveguide loaded by bare metallic cylinders, and (c) the measured reflections of the waveguides.

respectively. Figure 6(c) illustrates the reflection coefficients of the waveguides loaded by the coated and bare cylinders as function of frequency; it is found that the reflection coefficient of the array of the bare cylinders is about 0.8, while for the specially coated cylinders, the reflection coefficient is reduced to 0.15 at 9.0 GHz. The frequency shift from the predicted 8.2 GHz (predicted minimum TCS) to 9.0 GHz (measured minimum reflection) is believed to be due to the loss of the coating material. Nevertheless, the low reflection coefficient does indeed provide an evidence for the transparency of the designed coated cylinder. The coated cylinders act as vacuum and the bare cylinders act as an effective medium with a negative permittivity,[20,21] so the microwave is prohibited to propagate through the waveguide.

The scattering field of an object together with the superposition of the incident field will blur the wave front. However, a transparent object has no interference to the incident field. By scanning the total electric field of the coated and bare cylinders, the transparency of the coated cylinders will be clearly demonstrated. Using the electric field mapping system (similar to that in Ref. 3), the total field (the incident field plus the scattering field) of a cylinder and an array of four cylinders are scanned. The cylinders are placed into a chamber encircled by a microwave absorbing material between two parallel aluminum boards. *X*-band microwaves are sent out from one port of the vector network analyzer, and they are incident to the cylinders through a coaxial-waveguide adaptor at one edge of the chamber. A coaxial detector is placed on the upper aluminum board; it can be coupled with the electric field in the chamber and return the signals back to the vector network analyzer. The signal is transformed into S_{21} parameter by the analyzer. Then the electric field is depicted by the analyzer since its amplitude is linearly scaled to $|S_{21}|$. After calibration, the phase of the electric field can be considered as the same as the measured S_{21}. The lower board can be monitored to move by a motor, thus the detector can move relatively to the upper surface of the chamber and acquire the electric field at any position of the chamber. We first calibrate the zero-phase by putting the detector closest to the adaptor. Second, we let the detector "move" 0.1 by 0.1 mm as a step nearby the cylinder samples; therefore, the total electric field of the cylinders are measured.

According to the reflection property showed in Fig. 6, the actual operating frequency is selected as 9.0 GHz. In Fig. 7, the sources are located at the central point of the left side of each figures; the cylinders and coating layers are depicted by black dots and circles, respectively, for clarity. As shown in Fig. 7(a), the total field of a single copper cylinder without coating becomes disordered, especially for the field in the space between the source and the cylinder. Because the back-scattering waves overlap with the incident waves, analogous standing waves are formed in the space. To make the interference more clear, we place four copper cylinders by an array of $6.0\times6.0\ \text{mm}^2$ into the chamber. As shown in Fig. 7(b), a clear shadow is formed behind the cylinders. Replacing the bare copper cylinders by the carefully designed coated cylinders, the total electric fields are shown in Fig. 7(c) and 7(d), respectively. It is seen clearly that the incident

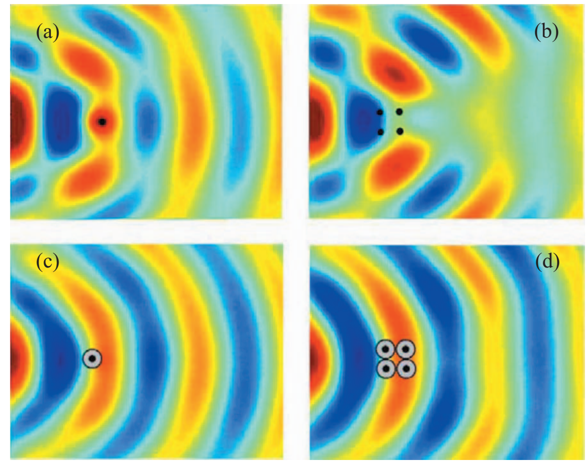

FIG. 7. (Color) Total fields of (a) a single bare metallic cylinder, (b) an array of four bare metallic cylinders, (c) a coated metallic cylinder, and (d) an array of four coated metallic cylinders.

wave can propagate almost freely forward as if there were no coated cylinders there. Since each of the coated cylinder is designed to be transparent, no matter how many coated cylinders are placed in the chamber, the total electric field will keep its pattern undisturbed as if the chamber were empty. This confirms our previous design of transparency for the coated metallic cylinders. Of cause the scatterings are also observed, which we attribute to the loss of the coating material; however, our experiments demonstrate clearly that the designed coated cylinders reduce dramatically the scattering.

IV. CONCLUSIONS

We have designed coated metallic cylinders, which are transparent to the EM wave. The method is based on the neutral inclusion method, i.e., the coated cylinders will be transparent and exhibit minimum scattering when they become "neutral inclusions" through proper coating. The effective permittivity of the coating cylinders is predicted by means of Mie scattering theory. Reflection coefficients of waveguides loaded with coated and bare cylinders are measured. The reflection coefficient of the coated cylinders is found to be much lower than that of the bare cylinders. Total scattering electric fields of the cylinders are also measured by an electric field mapping system. The results show that the coated cylinders become transparent while the bare cylinders scatter significantly the incident field.

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