Experimental study on transparency induced by metamaterials

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***Abstract*-With the deformation theory of continuum mechanics, we extend the coordinate transformation method to design arbitrary transformation metamaterials, which are inherently transparent for electromagnetic waves. In the experiment, the metamaterial capable of bending waves is verified. The transparency phenomenon based on the field cancellation effect is also discussed. The composites fabricated with Carbon nanotubes and rubbers are demonstrated experimentally to have the cloaking effect for metal cylinders.**

I. INTRODUCTION

Recently, the coordinate-transformation method [1] of Maxwell's equations is widely used to define materials with specific functionality, hardly realized by conventional mediums. According to this method, the material parameters are determined from the geometric transformation between the original and distorted spaces. Usually, the transformation mediums are inherently transparent and fabricated with metamaterial technology. This paper gives a new explanation of this method, which expresses the robustness in designing arbitrary transformation materials.

Another metamaterial induced transparency is based on the field cancellation effect [2]. Several works have been done to obtain the transparency conditions of coated/multilayered cylinders and spheres. In this work, we conduct some experiments of cloaking a metallic cylinder with isotropic dielectric materials.

II. ARBITRARY TRANSFORMATION METAMATERIALS

A. Design Theory

According to the coordinate transformation method, the permittivity ε' and permeability μ' in the transformed space are given by

$$\varepsilon' = \mathbf{A}\,\varepsilon\,\mathbf{A}^{\mathrm{T}}/\det\mathbf{A}, \tag{1a}$$
$$\mu' = \mathbf{A}\,\mu\,\mathbf{A}^{\mathrm{T}}/\det\mathbf{A}, \tag{1b}$$

where ε and μ are the permittivity and permeability in the original space. $\mathbf{A}$ is the Jacobian transformation tensor with components $A_{ij}=\partial x'_i/\partial x_j$.

In the continuum mechanics [3], the tensor $\mathbf{A}$ denotes the deformation gradient for an infinitesimal element $d\mathbf{x}$ deformed to $d\mathbf{x}'$ under the space transformation. The deformation can be decomposed into pure stretches (described by a positive definite symmetric tensor $\mathbf{V}$) and a rigid rotation (described by a proper orthogonal tensor $\mathbf{R}$), so the tensor $\mathbf{A}$ can be expressed as $\mathbf{A}=\mathbf{VR}$. Suppose material parameters of original materials are ε_0 and μ_0. Using the relation $\mathbf{B}=\mathbf{FF}^{\mathrm{T}}=\mathbf{V}^2$ and $\mathbf{R}^{\mathrm{T}}\mathbf{R}=\mathrm{I}$ ($\mathbf{I}$ is the unit tensor), from equation (1) we can obtain

$$\varepsilon' = \varepsilon_0\,\mathbf{B}/\det\mathbf{A}, \tag{2a}$$
$$\mu' = \mu_0\,\mathbf{B}/\det\mathbf{A}. \tag{2b}$$

In the principal system, the tensor $\mathbf{B}$ only has diagonal components $\mathrm{B}=\mathrm{diag}[\lambda_1^2, \lambda_2^2, \lambda_3^2]$, where λ_i (i=1,2,3) are the eigenvalues of the tensor $\mathbf{V}$, which characterize the principal stretches of an infinitesimal element. Combining with $\det\mathbf{A}=\lambda_1\lambda_2\lambda_3$, we can rewrite equation (2) as

$$\varepsilon' = \varepsilon_0\,\mathrm{diag}[\lambda_1/\lambda_2\lambda_3, \lambda_2/\lambda_1\lambda_3, \lambda_3/\lambda_1\lambda_2], \tag{3a}$$
$$\mu' = \mu_0\,\mathrm{diag}[\lambda_1/\lambda_2\lambda_3, \lambda_2/\lambda_1\lambda_3, \lambda_3/\lambda_1\lambda_2]. \tag{3b}$$

Equation (3) is an alternative explanation of the transformation method (1). It means that the material parameters in the transformed space can be calculated by the principal stretches of deformation gradient tensor $\mathbf{A}$ during the space transformation. It is noted that the rigid rotation has no contribution on the final material parameters. The proposed method is easily used to design a cloak with arbitrary shapes. Suppose that a cloak has the inner boundary a and outer boundary b. In the boundary value problem of continuous mechanics, keep the outer boundary fixed and let a=0. Using Laplace's equations $\Delta\mathbf{x}=0$, we can solve the deformation fields uniquely, which in turn employed to determine the material parameters of the random cloak with help of equation (3). Figure 1 gives simulation results of a random cloak designed with the proposed method.

B. Experimental Fabrication

Using the transformation method, we design a metamaterial capable of bending propagation directions of electromagnetic waves. The original and transformed spaces are shown in figure 2. The nonzero components of the permittivity and permeability tensor ε and μ of the transformation media are finally derived from (3) as

$$\varepsilon_r = \mu_r = 2a/(r\pi), \tag{4a}$$
$$\varepsilon_\theta = \mu_\theta = r\pi/(2a). \tag{4b}$$
$$\varepsilon_z = \mu_z = 2a/(r\pi). \tag{4c}$$

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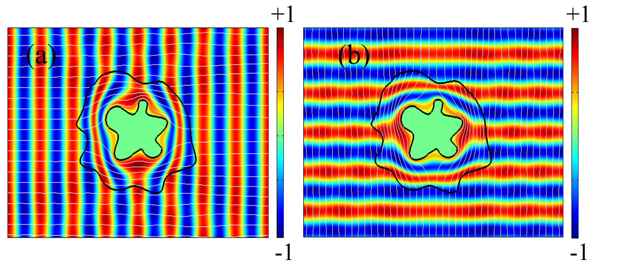

Figure 1. Electric field distributions of an arbitrary cloak illuminated by (a) horizontally and (b) vertically incident plane waves. (The white lines indicate the flow of electromagnetic power)

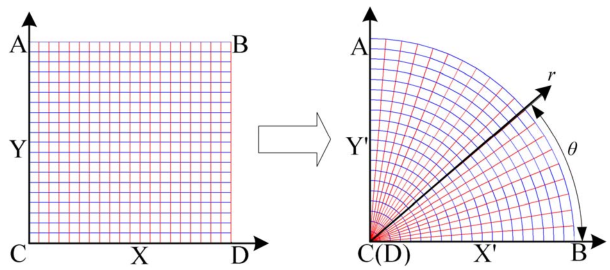

Figure 2. The scheme of original and transformed spaces

The single split ring is employed to realize the simplified material parameters $\varepsilon_z=2$, $\mu_r=2a^2/(r\pi)^2$, and $\mu_\theta=1/2$. By adjusting the micro-structural parameters of the split-ring resonators, we fabricate the bending metamaterials shown in figure 3.

To examine the functionality of the transformation metamaterials, we have built up an electric-field mapping system shown in figure 4, which is similar to the setup in Ref. [4]. The electric fields measured on the surface of metamaterial are shown in figure 5 and compared with simulation results of materials with ideal parameters. It is seen that the fabricated metamaterials indeed bend wave propagation, although not as the desired way. This may be induced by the simplified parameters of the metamaterial as well as the fabrication error.

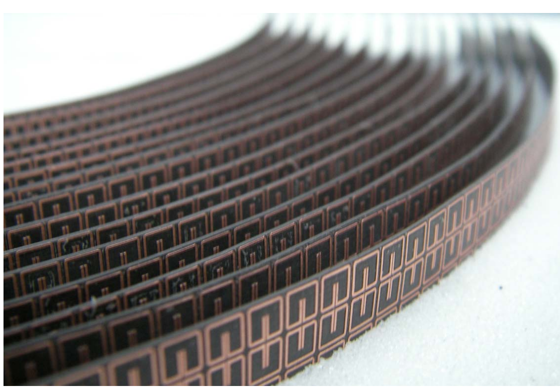
Figure 3. The fabricated bending metamaterial

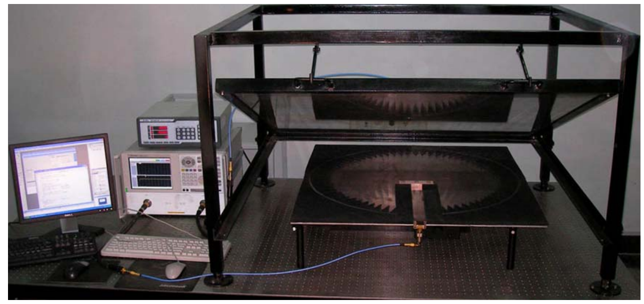
Figure 4. The electric-field scanning system

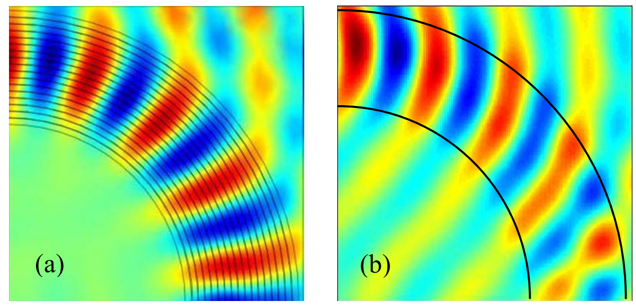

Figure 5. (a) The simulation and (b) measured results of electric field distributions of the bending metamaterials.

III. Plasmon Induced Transparency

According to the concept of neutral inclusion [5], composites containing small scatterers will become transparent if their effective material parameters equal those of the embedded matrix. With this idea, metal cylinders coated by regular dielectric material could be made transparent, since periodically arranged metal cylinders can respond to be of negative polarization [6] and dielectric coatings have positive polarization. Then the effective permittivity can be same as that of the air.

When an infinite long cylinder coated by a dielectric material is incident by a plane wave with the electric field polarized along the axis of the cylinder, the total scattering cross section of the cylinder can be calculated as [7]

$$\sigma_{\text{total}} = \frac{4}{k_1}\sum_{-\infty}^{\infty} A_n A_m^*, \tag{5}$$

where k_1 is the wave vector in the matrix and the superscript * means the conjugate of A_m. The scattering coefficient A_m can be readily computed by matching the electric and magnetic field at the boundary. For a cylinder of radius a with a coating of outer radius b, the scattering coefficient A_n is written as

$$A_n = \frac{k_2 J_n(k_1 b) G_n - k_1 J_n'(k_1 b) M_n}{-k_2 H_n^{(1)}(k_1 b) G_n + k_1 H_n''^{(1)}(k_1 b) M_n} \tag{6}$$

where

$$G_n = H_n'^{(1)}(k_2 b)H_n^{(2)}(k_2 a) - H_n'^{(2)}(k_2 b)H_n^{(1)}(k_2 a)\,,$$
$$M_n = H_n^{(1)}(k_2 b)H_n^{(2)}(k_2 a) - H_n^{(2)}(k_2 b)H_n^{(1)}(k_2 a)\,.$$

The prime in the superscript means the derivative with respect to the argument. J_n and N_n are the Bessel functions of the first and second kinds, $H_n^{(1)} = J_n + i\,N_n$, $H_n^{(2)} = J_n - i\,N_n$, and k_2 is the wave vector in the coating.

Figure 6 shows the total cross sections of a bare metal cylinder of radius 1*mm* and that coated by a dielectric material of permittivity 14 and radius 3*mm*. It is seen that the total scattering cross section will increases with increasing of the frequency when the radius of the metal cylinder is much small than the operating wavelength. In addition, it is found that the cover material has the cancellation effect for electric fields scattered by the metal cylinder. At the frequency around 8.5 GHz, the composite cylinder has much small total scattering cross section than that of the bare cylinder, giving rise to the transparency phenomenon.

We then conduct some experiments to verify this transparency phenomenon. In the experiment, the coating material is chosen as the mixtures of Carbon nanotubes (CNT) and rubber, by a mixing (mass) ratio about 60:100. After the hot-press process, the composite exhibits a high dielectric constant. The dielectric constant of the composite is dependent on the fraction of the nanotubes. Figure 7 shows the dielectric constants of the composites with different fractions of the nanotubes.

The composites are then employed to be the cover materials of metal cylinders. According to the designed parameters, the cover has the inner radius 1 *mm* and outer radius 3 *mm*. Using a waveguide, we have measured the reflectance of the bare and coated cylinder clusters. Figures 8 (a) and (b) show the waveguides loaded by five coated metallic cylinders and bare cylinders. Figure 8 (c) plots the power reflection of both materials. It is found that the reflection is reduced evidently around 9 GHz, very close to the predicting frequency in figure 6.

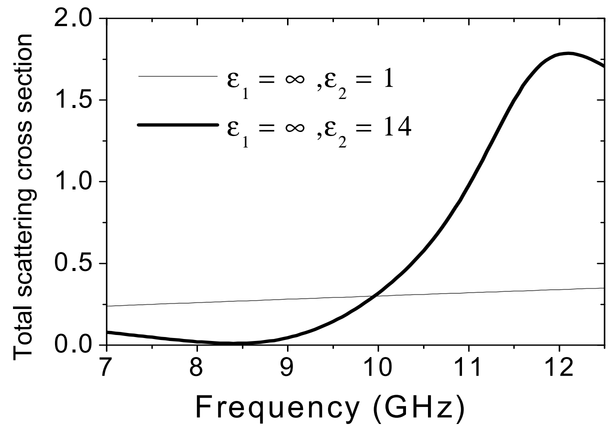

Figure 6. The total cross section of a cylinder coated by materials with different dielectric parameters.

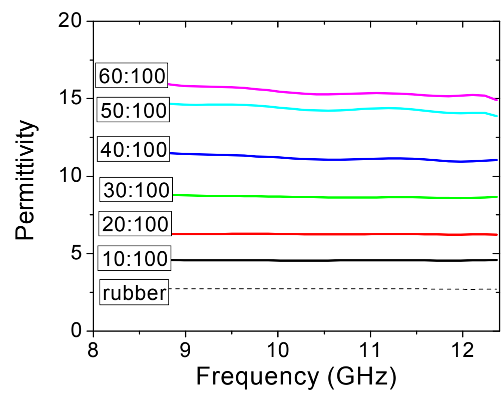

Figure 7. The dielectric constants of the composites made from Carbon nanotubes and rubber with different mixing ratio.

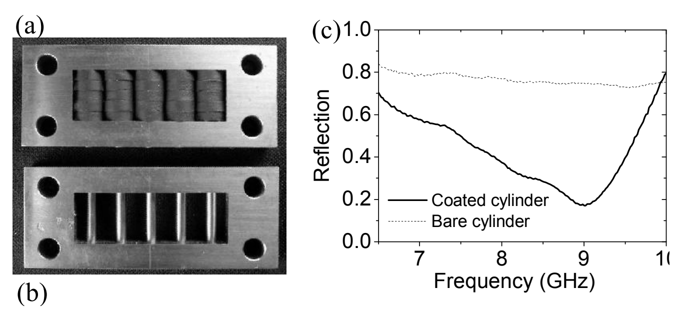

Figure 8. (a) Waveguide loaded by coated cylinder, (b) waveguide loaded by metallic cylinder, (c) the reflections measured in the waveguides.

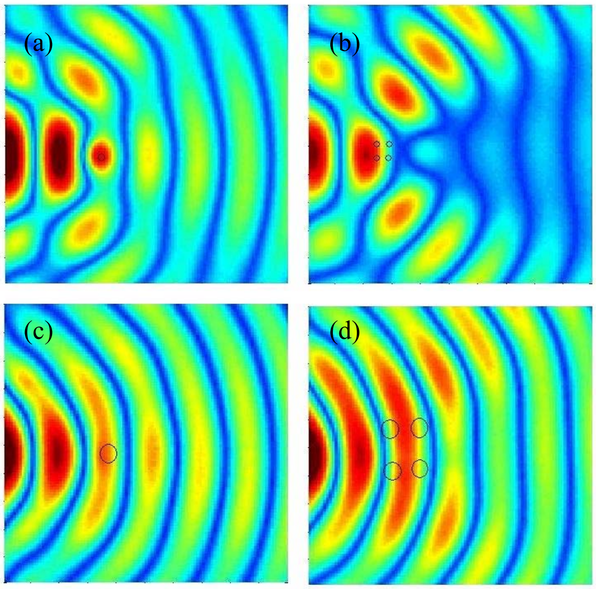

Figure 9. The measured electric fields of (a) a bare metal cylinder, (b) an array of bare cylinders, (c) a coated cylinder, and (d) an array of coated cylinders.

Using the electric-field mapping system shown in figure 4, we measure the electric fields scattered by a single and an array of metal cylinders at the frequency 9.0 GHz, as shown in figures 9 (a) and (b). It can be seen that evident shadows are produced in the forward-scattering area. However, the coated cylinders have very small scatterings, exhibiting clearly the transparency phenomenon.

IV. Conclusions

In this work, we discuss transparency phenomena based on two different mechanisms, the coordinate transformation and field cancellation effect. The transformation method has been extended to design arbitrary transformation materials with help of the deformation theory. The metamaterial with bending wave property is verified in an experiment. The composites filled with Carbon nanotubes and rubbers are expected to have cancellation effects for scatterings by metal cylinders. In the experiments, the cloaking effect of the composite are finally verified.

Acknowledgment

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References

[1] J.B. Pendry, S. Schurig, and D.R. Smith, "Controlling electromagnetic Fields," *Science,* vol. 312, pp. 1780-1782, June 2006.

[2] A. Alu and N. Engheta, "Achieving transparency with plasmonic and metamaterial coatings," *Phys. Rev. E,* vol. 72, pp. 016623, July 2005.

[3] W.M. Lai, D. Rubin and E. Krempl, *Introduction to Continuum Mechanics*, 3rd ed., Butterworth-Heinemann, Burlington, 1995.

[4] D. Schurig, J.J. Mock, B.J. Justice, S.A. Cummer, J.B. Pendry, A.F. Starr, and D.R. Smith, "Metamaterial Electromagnetic Cloak at Microwave Frequencies," *Science,* vol. 314, pp. 997-980, November 2006.

[5] X.M. Zhou and G.K. Hu, "Design for electromagnetic wave transparency with metamaterials," *Phys. Rev. E*, vol. 74, pp. 026607, August 2006.

[6] J.B. Pendry, A.J. Holden, and W.K. Stewart, "Extremely low frequency plasmons in metallic mesostructures," *Phys. Rev. Lett.*, vol. 76, pp. 4773-4776, December 1996.

[7] C.C. Tang, "Backscattering from dielectric-coated infinite cylindrical obstacles," *J. Appl. Phys.*, vol. 28, pp. 628-633, November 1956.