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Compact acoustic double negative metamaterial based on coexisting local resonances

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Acoustic metamaterials generally exploit resonances to assume negative properties. While many types of resonances can be used for achieving a negative bulk modulus, the number of types of resonances for generating a negative mass density is limited. In this work, a double negative airborne acoustic metamaterial is proposed, whose negative density is achieved from Fabry-Pérot resonance. More specifically, each unit cell of the metamaterial comprises only a single element, allowing coexistence of local Helmholtz resonance and Fabry-Pérot resonance to simultaneously generate a negative modulus and negative density. The metamaterial exhibits a reversed phase velocity and negative refraction, even if the constitutional elements are randomly distributed. This is a pioneering work that an airborne acoustic double negative metamaterial derives negative density from Fabry-Pérot resonance and maintains negative refraction with its constitutional elements randomly distributed. Published by AIP Publishing. https://doi.org/10.1063/1.5052026

The versatile capabilities of acoustic metamaterials,1–29 and their electromagnetic counterparts,30–35 in wave controlling have enabled a wide variety of anomalous applications, such as negative refraction,8–12 super-resolution imaging,3,12 cloaking,12,28 and perfect absorption.2,20 These versatile capabilities arise from the formation of metamaterials’ internal architecture, rather than merely from their composition. Accordingly, it offers scientists greater agility to impart a material with desired collective functions, such as a simultaneously negative modulus and density, by simply tailoring the feature elements or unit cells. So far, a number of versions of double negative metamaterials (DNMs) have been proposed in acoustics.8–13,22,23,25,27 However, the types of DNMs in airborne acoustics, the most common acoustic branch, remain limited and still lack practicability. Generally, the airborne DNMs rely on either resonance or spatial modulation25 to assume negative parameters. The number of types of resonances that can be made use of is actually limited, which mainly includes membrane resonance,1,2,16 Helmholtz resonance,4,10 and more recently Mie resonance.7 In airborne acoustics, while both the Helmholtz resonance and Mie resonance can be used to generate a negative bulk modulus, until now, the membrane resonance is probably the only way to obtain negative density. The other approach of achieving a DNM from spatial modulation25 would also have certain limitations, because the “spatial modulation” DNM is comprised of maze-like curled channels arranged with a predetermined lattice constant, so that the lattice constant affects the length of the channels and accordingly affects the “DNM” effective properties. Therefore, it is advantageous to propose an alternative type of resonance to generate a negative density, to improve the adaptability and practicability of the DNM.

In this paper, we report a design of an airborne acoustic DNM based on coexisting local resonances, namely Helmholtz resonance and Fabry-Pérot (FP) resonance. While the Helmholtz resonance is well knowledgeable in the field of metamaterials, and the FP resonance associated with waveguides of an abruptly reduced cross-section has also been used to achieve deep-subwavelength super-imaging,2 the use of Fabry-Pérot resonators (FPRs) as localized elements for generating a negative density has never been demonstrated. Here, we show that a potential advantage of the FPR is the FP resonance can coexist with the Helmholtz resonance, based on which the Helmholtz resonator (HR) and the FPR can be merged into one single localized element.

As shown in Fig. 1, the unit cell of the proposed DNM comprises only a single element [Fig. 1(d)], adapted from a traditional Chinese Taiji logo [Fig. 1(a)]. By twisting the S-like outline to form two isolated cavities, the outline of the Taiji logo can be tailored to define a pair of HRs. As shown in Fig. 1(b), each cavity of the HRs holds a volume of air in communication with the outside air medium through an aperture, created by the ends of the “S” outline and its opposing intermediate portion. The volume of the cavity and the dimension of the aperture determine the resonant frequency of the HRs. If an array of the twin-resonators is arranged in an air matrix, a single negative metamaterial assuming a negative bulk modulus can be readily created. Now referring to Fig. 1(c), the outline of the Taiji is replaced by a double-layer twisted “S” wall, with an elongated channel formed in-between, while the wall now defines a pair of sealed cavities, without apertures to the outside. The elongated channel ends with a pair of opposite openings, through which the air inside the channel communicates with the outside air medium. This forms abruption in the cross-sections of the airflow in and out of the channel at the openings, giving rise to an FP

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The frequency of the FP resonance is mainly determined by the length of the channel, i.e., \( ql = m\pi \), where \( q \) is the effective wave number in the channel.\(^{36}\) \( l \) is the length of the channel, and \( m \) is an integer equal to 1 in this study.

By coupling the HR of Fig. 1(b) and the FPR of Fig. 1(c), an integrated element can be created, as shown in Fig. 1(d). It will be shown that this configuration allows the coupled element to support the coexistence of two localized resonances.

The transmission properties of HRs, FPRs, and the coupled elements are calculated by using Comsol Multiphysics and measured experimentally. The experimental setup for the transmission test is shown in Fig. 2(a). A rigid plastic rectangular waveguide is fabricated and connected to a tube sound source (BJSW SW230). A rear rectangular waveguide filled with sound absorber (glass wool) is connected to the waveguide through a 3D printed microphone fixture. A row of 4 HRs, FPRs and the coupled elements are placed in the waveguide respectively, so the microphone measures the transmission amplitudes while the sound absorber creates a perfect match layer (PML). For the construction of the structures in Fig. 1, the center line (not shown) of the channel is defined by equations of \( x = r_0 \cdot g(s)/(2s + n_0) \cdot \cos(s) \) and \( y = -r_0 \cdot g(s)/(2s + n_0) \cdot \sin(s) \), in which \( r_0 = 5.5 \) mm, \( 0 < s < s_0 \), and \( s_0 = 3\pi/2 \). These equations are derived from data fitting the outline of the Taiji logo onto an Archimedean spiral with end correction. The variable \( s \) corresponds to the number of turns of the spiral-like outline, \( r_0 \) corresponds to the radius variation when \( s \) increases by \( 2\pi \), and the term \( g(s)/(2s + n_0) \) inwardly converges the end portion of the outline towards the center starting point. The selected parameters give the S-channel a length of \( l = 86.9 \) mm, a width of \( 1.0 \) mm, and a working frequency of around 2000 Hz. The size of the unit cell [square in Fig. 1(d)] is \( a = 28 \) mm. In simulation, the waveguide is specified with “Sound hard Boundary,” and the S-walls of the HRs, FPRs and the coupled elements are modeled by the “Interior Sound Hard Wall,” without thickness. In experiment, the actual thickness of the S-walls is 0.25 mm.

The effective density and modulus of the element are retrieved by using an integrated-transfer matrix method\(^{37}\) based on the simulated acoustic pressure and velocity data. First, the relation between acoustic pressures and velocities at the opposite sides of the unit cell containing the coupled element can be established as

\[
\begin{bmatrix}
p_x \\
u_x
\end{bmatrix} =
\begin{bmatrix}
\cos(k_{\text{eff}}a) & jZ_{\text{eff}}\sin(k_{\text{eff}}a) \\
\frac{1}{j} \sin(k_{\text{eff}}a)/Z_{\text{eff}} & \cos(k_{\text{eff}}a)
\end{bmatrix}
\begin{bmatrix}
p_b \\
u_b
\end{bmatrix},
\]

where \( k_{\text{eff}} \) and \( Z_{\text{eff}} \) are the effective wavenumber and impedance of the unit cell to be retrieved, \( p_x, p_b \) are pressures, \( u_x, u_b \) are particle velocities obtained in simulation, and \( x, \beta \) represent the left and right side of the unit cell. Therefore, the effective density and modulus of the unit cell can be determined by\(^{12,38,39}\)

\[
\rho_{\text{eff}} = Z_{\text{eff}} k_{\text{eff}} / \alpha, \quad K_{\text{eff}} = \omega Z_{\text{eff}} / k_{\text{eff}},
\]

in which \( Z_{\text{eff}} = (p_x^2 - p_b^2)/(u_x^2 - u_b^2) \) and \( \cos(k_{\text{eff}}a) = (u_x p_x + u_b p_b)/(u_x p_x + u_b p_b) \).

As shown in Fig. 2(b), the HR generates a wide sound stop-band between 1550 Hz and 2450 Hz, while the FPR generates a narrow stop-band between 1800 Hz and 1950 Hz, rightly falling in the wide stop-band. The wide range of the stop-band is mainly induced by the comparably large filling ratio of the pair of HR elements to the unit cell. Consequently, an advantage of the Taiji logo is that the two types of resonances inherently overlap in the frequency range, without requiring extensive adjustments to the structural parameters.

The calculated and measured transmission properties of the coupled elements of Fig. 1(d) are shown in Figs. 2(c) and 2(d). Comparing the lines in Figs. 2(b) and 2(c), it can be
seen that the overlapping range of the stop-bands has now reversed into a pass-band in the region of 1750 Hz and 1900 Hz, while the transmission at the rest part of the spectrum generally remains unchanged. This is because that, in this pass-band, the coupled element turns into an acoustic DNM with a simultaneous negative modulus and density. As illustrated in Fig. 2(e), the effective modulus $K_{\text{eff}}$ of the HRs and the effective density $\rho_{\text{eff}}$ of the FPRs are negative in the range of 1600–2200 Hz and 1750–1850 Hz, respectively, so the combined elements take double negativity and a real wavenumber ($k_{\text{eff}} = -\sqrt{-\rho_{\text{eff}} K_{\text{eff}}} = n_{\text{eff}} k_0$, with $k_0$ being the wavenumber of air) in the overlapping range of 1800–1900 Hz (with a small shift of 50 Hz). Note that if only one of $K_{\text{eff}}$ or $\rho_{\text{eff}}$ is negative, the wavenumber takes an imaginary value. As a further explanation, the band structure of the combined element is also obtained and shown in Fig. S3. It can be found that, in the range of 1800–1900 Hz, the band of the combined element takes a reverse slope (triangles) as compared with the first and third positive pass-band (circles and squares). From Figs. 2(c) and 2(d), it can also be seen that the numerical result agrees well with the measured result.

To reveal the underlying physical mechanism of the “double negativity” of the metamaterial, the distribution of the acoustic field at 1850 Hz is obtained and shown in Fig. 3(a). The mechanism of negative density will first be discussed. Illustratively, the effective density can be defined as $\rho_{\text{eff}} = -\nabla p/(\dot{u})$ (Ref. 40) according to the Cauchy momentum equation, with $\dot{u}$ signifying the average particle acceleration over the unit cell and in the 2D case $\nabla p = (p_x - p_y)/a$. The inset of Fig. 3(a) indicates that, at a given moment, $p_x < p_y$. However, based on the distribution of acceleration shown in Fig. 3(a), it can be observed and calculated that the resultant particle acceleration is $\langle \dot{u} \rangle < 0$. Therefore, the effective density $\rho_{\text{eff}} < 0$. Similarly, the effective modulus can be defined as $K_{\text{eff}} = -V \partial \rho / \partial V$,\textsuperscript{40} where $V = a^2$ is the volume of the unite cell. The derivation of $\partial V$ can be equivalently treated as the amount of air flowing into or out from the cell; $\partial V = a \cdot u$, and the $\partial p$ signifies the fluctuation of the acoustic pressure. As can be seen from Fig. 3(b), at the moment the pressure is falling down [as it reaches maximum, inset of Fig. 3(a)], which should result in an outflow if $K_{\text{eff}} > 0$. However, due to the resonance of the coupled element, the air in the top cavity flows out from the HR (arrow 1), while the air in the bottom cavity flows into the HR (arrows 2 and 6). Simultaneously, the air outside the channel moves into the channel from both end openings (arrows 5 and 7). This makes a net amount of air to flow into the channel of the FPR (arrows 5 and 7). As a result, the amount of air in the unit cell but outside the element decreases, which causes inflows from both sides of the unit cell.

A direct consequence of double negativity is that the phase delay along the route of the sound wave inside the DNM is negative.\textsuperscript{41} Figure 3(c) shows the phase delay pattern of the sound wave in and near the DNM at 1850 Hz. Assuming the sound wave is a plane wave, the phase delay can be briefly calculated as: $\theta = k \cdot x - \omega \cdot t$, where $k$ is the wave number, and $k = k_0$ in the air, and $k = k_{\text{eff}}$ in the DNM. Given the frequency of 1850 Hz, it can be determined from Fig. 3(c) that after a distance of $x = 0.18$ mm, the phase delay in air is $\theta = 2 \pi$, with the pressure equal to that of $\theta = 0$ when $x = 0$. Similarly, a contour of equiphase can be depicted. As shown in Fig. 3(c), the contours of the equiphases in the air medium upstream the DNM (left part) and downstream the DNM (right part) both show positive slopes, signifying $k = k_0 > 0$. By contrast, the equiphases in the DNM (middle part) show negative slopes, signifying $k = k_{\text{eff}} < 0$. The absolute values of the sloping rates of the equiphase contour are approximately equal, indicating that the DNM has a refractive index around $-1$, or $k_{\text{eff}} \approx -k_0$. The negative slope of the equiphase contour of the DNM provides straightforward and intuitive evidence that the Taiji structure is a DNM.

Through simulation and experiment, now we demonstrate that negative refraction can be realized by the DNM, with the coupled elements either periodically or randomly distributed. To this end, we first fabricated a wedge-shape sample containing 30 unit cells of the coupled elements, placed in a chamber between two parallel boards flanked by sound absorbers, as shown in Fig. 4(a). The wedge has a right-angled triangle with a hypotenuse “1” about 71.6° angle relative to a bottom side in the horizontal direction. A horn loudspeaker emits directional sound waves towards the sample and a 2D movable microphone is arranged downstream the wedge-sample to measure the refractive field. The refractive field is also numerically simulated for comparison. The incident sound waves “I” are horizontally impinged onto the left side of the wedge and penetrate into the wedge until reaching with the hypotenuse “1.” The simulated pressure field distribution of the DNM at 1850 Hz shows that the transmitted sound waves “T” are about 18° up away from the norm “2,” situated in the same side of the norm “2” with

![Image](63x87 to 285x340)

FIG. 3. (a) The particle acceleration distribution of the combined element at 1850, (b) the particle velocity distribution of the coupled element at 1850 Hz, and (c) the phase delay pattern in the air medium and coupled elements.
the incident sound waves “I.” This illustrates that the refractive angle of the transmitted sound waves of this DNM wedge is inverted, as compared with that of the regular medium. Therefore, the refractive index of the DNM is negative, and $n_{\text{eff}} \approx -1$ is assumed, given the fact that $\theta_{\text{in}} \approx \theta_{T}$. An inset illustrating the measured partial pressure distribution of the same DNM at the same frequency is well superposed in Fig. 4(b). Figure 4(c) further shows the simulated and measured negative refraction field of the DNM, with the coupled elements randomly distributed to form a wedge-sample. The random distribution of the elements is realized by assigning each element with arbitrary $x$- and $y$-coordinate values. As can be seen, no striking field differences exist for the case of randomly distributed elements in Fig. 4(c) as compared with those in Fig. 4(b). It is worth noting that the Taiji elements in Fig. 4(c) are also randomly rotated.

In conclusion, by using the local resonant FPR to generate a negative density and merging the FPR with the HR, this work proposes an airborne DNM, comprising coexisting localized resonant elements. Achieving an airborne acoustic negative density from FP resonance not only increases the number of options of the acoustic metamaterial design, but also allows the proposed metamaterial to be more compact and to be independent of the cell arrangement. Thus, the DNM demonstrates nearly invariable refractive properties even when the elements are randomly distributed and rotated, which enables the flexibility in deploying the DNM, or the constitutional elements themselves for various acoustic wave manipulation and modulation purposes.

See supplementary material for the picture of the experimental setup for measuring the refractive field of the double negative metamaterial and the band structure of the double negative metamaterial with a negative slope.

\footnotesize{FIG. 4. (a) The experimental setup for measuring the acoustic pressure field of the double negative metamaterial. The simulated and measured negative refraction properties of the wedge-sample of the DNM at 1850 Hz, with the transmitted waves and the incident waves situated at the same side of the norm, with (b) periodic arrangement, and (c) random arrangement, and the insets show the experimentally measured partial pressure field distribution.}