Tunable network sound absorber based on additive manufacturing\textsuperscript{a)}

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I. INTRODUCTION

Noise reduction is necessary for human health in many working and living scenarios. Conventional sound absorbers, such as porous and fibrous materials (Allard and Atalla, 2009; Biot, 1956; Huang et al., 2014), generally require a structural thickness comparable to wavelength, which results in bulky structures for low-frequency noises. Micro-perforated panels with backed cavities can absorb low-frequency sound efficiently but still suffer from a large thickness of the back cavity (Maa, 1975, 1998; Toyoda et al., 2011; Wu et al., 2019). During recent decades, acoustic metamaterials (Fang et al., 2006; Liu et al., 2000; Huang and Sun, 2010) and metasurfaces (Li et al., 2015; Xie et al., 2014) with a sub-wavelength thickness have shown tremendous functionalities of acoustic wave control, such as negative refraction (Kaina et al., 2015; Liang and Li, 2012; Xie et al., 2013), deep sub-wavelength focusing/imaging (Christensen and Javier García de Abajo, 2012; Li et al., 2009; Zhu et al., 2011), and cloaking (Chen et al., 2017; Popa et al., 2011). Particularly, sound-absorptive metamaterials consisting of resonant units, such as membrane-type resonators (Ma et al., 2014; Mei et al., 2012), Helmholtz resonators (Cai et al., 2014; Chen et al., 2017; Ryoo and Jeon, 2018), and Fabry–Pérot (FP) channels (Jiang et al., 2014; Yang et al., 2017), can realize even high sound absorption at targeted low frequencies. However, due to the dispersive nature of the resonances, narrowband performance largely hinders their applications.

To obtain broadband absorption, one feasible way is to combine several detuned components together. Yang et al. (2017) used curled FP channels to form an optimal sound absorption structure. The components of the structure are generally arranged in parallel, so it is difficult to assemble the channels with gradient lengths into a compact and thin absorber (Huang et al., 2018). Designing tube networks inside a cubic can circumvent this problem (Cambonie and Gourdon, 2018, van der Eerden, 2000). However, the tube network absorber can only work well at fixed frequencies once fabricated. Tunable sound-absorbing metamaterials have been realized by embedding functional structures into metamaterials, such as apertures (Huang et al., 2019), Mie resonators (Shao et al., 2021), and panels (Wang et al., 2018). Additive manufacturing techniques are suitable for the implementation of tunable sound-absorbing metamaterials (Cavaliere et al., 2021; Ngo et al., 2018; Zhang et al., 2016). Here, we propose a tunable sound absorber consisting of optimized tube networks based on additive manufacturing. The key to achieving tunability is to decompose a generalized tube network into two types of elements: honeycomb plates and chips, as shown in Fig. 1. The honeycomb panels consist of multiple identical tubes arranged in parallel, of which the length is equal to the thickness of the honeycomb panels. The chips can control the opening and closing of each tube in the honeycomb plate as well as the...
connectivity of adjacent tubes in the honeycomb plate. Given fixed honeycomb plates, we tune the sound absorption performance of the network absorber by simply changing its chips. The low reduced frequency model and genetic algorithm (GA) are used to design the chips, according to targeted absorption spectrums. We design different groups of chips, which are fabricated by three-dimensional (3D) printing technology, and experimentally validate the targeted low-frequency sound absorption spectrums. Based on the optimized network absorber, we realize broadband and low-frequency sound absorption. The remarkable efficiency and versatility of tunable network sound absorbers may pave the way for programmed absorbing material design.

The paper is organized as follows: In Sec. II, a theoretical model for sound absorption of a tube network and numerical algorithm for optimizing the tube distribution are introduced. The theoretical model is validated through experiments. Section III is concerned with designing network absorbers according to various optimizing targets, including broadband absorption and specific absorption at specified frequency ranges. The low-frequency sound absorption performance of the tunable absorber is experimentally validated in Sec. III. The impact of the radius of tubes on absorption and the 3D structure of the network are discussed at the end of Sec. III. The main findings and conclusions are finally summarized in Sec. IV.

II. METHODS

A. Theory

Sound absorption in a tube originated from thermal and viscous effects of air can be characterized by a low reduced frequency model (Tijdeman, 1975; Zwicker and Kosten, 1949), which is briefly recalled in the following. Consider a tube with length \( l \) [Fig. 2(a)], of which two ends are denoted as \( i \) and \( j \). According to the low reduced frequency model, acoustic pressure \( p(x) \) and velocity \( v(x) \) at location \( x \) in the tube [see Fig. 2(a)] with circular frequency \( \omega \) can be analytically described,

\[
p(x) = p_A \exp(\Gamma k_0 x) + p_B \exp(-\Gamma k_0 x),
\]

\[
v(x) = \frac{G}{\rho_0 c_0} (p_A \exp(\Gamma k_0 x) - p_B \exp(-\Gamma k_0 x)),
\]

in which \( p_A \) and \( p_B \) can be regarded as the incident and reflection coefficient, \( \rho_0 \) and \( c_0 \) denote the density and sound velocity of air, respectively, and \( k_0 = \omega/c_0 \) is the wave number. Parameters \( G \) and \( \Gamma \) are defined as

\[
G = -i\gamma/(\Gamma n), \quad \Gamma = \sqrt{\frac{J_0(i\sqrt{s})\gamma}{J_2(i\sqrt{s})\gamma}},
\]

\[
n = \left( 1 + \frac{(\gamma - 1)J_2(i\sqrt{\sigma})\gamma}{J_0(i\sqrt{\sigma})\gamma} \right)^{-1},
\]

where \( J_0 \) and \( J_2 \) are Bessel functions of the first kind of order 0 and 2, respectively. Parameter \( s = R(\rho_0 \omega^2)/\mu \) is the shear wave number, with \( \mu \) denoting the dynamic viscosity (18.2 \( \times 10^{-6} \text{ Ns/m}^2 \)); the square root of the Prandtl number \( \sigma \) is 0.84, and the specific heat \( \gamma \) is 1.4. \( R \) denotes the radius of the tubes.

Mass flow disturbances at both ends of the tubes are \( Q_i = \rho_0 A v_i \) and \( Q_j = -\rho_0 A v_j \), where \( v_i \) and \( v_j \) are the acoustic velocity at both ends, and \( A \) is the cross section area of the tube. With the pressure and velocity distributions in Eqs. (1) and (2), the two mass flow disturbances are related to pressures at both the ends,

\[
\begin{pmatrix}
Q_i \\
Q_j
\end{pmatrix} = \frac{AG}{c_0 \sinh(\Gamma k_0 l)} \begin{pmatrix}
-cosh(\Gamma k_0 l) & 1 \\
1 & -cosh(\Gamma k_0 l)
\end{pmatrix} \begin{pmatrix}
p_i \\
p_j
\end{pmatrix},
\]

in which \( p_i \) and \( p_j \) are the pressure disturbances at the two ends. The low reduced frequency model is generally expressed in terms of such a transmission relation. If the two end pressures are known, the pressure and velocity distributions in the tube can be obtained.

To verify the accuracy of the low reduced frequency model, we calculate the sound absorption performance of a cylinder tube using the narrow region acoustics of the pressure acoustics module in COMSOL Multiphysics® 5.5. We used three-node plane solid elements. As shown in Fig. 2(e), the tube ends up with a hard wall, of which the length is 200 mm and the diameter is 5 mm. The surface porosity of the tube is 1/8. The numerical result agrees well with the theoretical result.

Figure 2(b) shows a tube network, in which every horizontal and vertical line represents a tube. The network has \((m-1) \times n \) nodes and \(2mn - m - 3n + 2 \) tubes. To calculate the sound absorption of the tube network, a plane wave is incident from the left side. Under low-frequency
conditions, pressures at the entrance of the tubes are the same and supposed to be unit \( p_k = 1 \text{ Pa} \) \((k = 1, 2... n)\). The pressures at other tube ends are called node pressure. The tube network has in total \((m - 1) \times n\) unknown node pressures \( p_k \) \((k = n + 1, n + 2... \text{nm})\). For each node connecting more than one tube as illustrated in Fig. 2(c), a conservation law holds for the mass flow disturbance,

\[
\sum_{k=1}^{4} Q_k + \frac{f_l k_0}{c_0 n} p_0 V = 0.
\]

(5)

\( Q_k \) can be obtained from two node pressures of each tube, and \( V = 16 R^3 / 3 \) is the intersecting volume of multiple circular tubes. We can construct a linear equation in terms of node pressure \( p_k \) for each node, and \((m - 2) \times n\) equations can be obtained. For tubes terminated with the sound hard wall shown in Fig. 2(d), the mass flow disturbance at the right side of the tube is zero,

\[
Q_6 = p_5 - \cos(\Gamma k_0) p_6 = 0.
\]

(6)

In total, \( n \) equations for zero mass flow can be obtained for the above model. Now the number of equations and the number of unknown node pressures are the same, \((m - 1) \times n\). These equations can be assembled into a matrix equation,

\[
M(p_{n+1}, p_{n+2}, \ldots, p_{nm})^T = f(p_1, p_2, \ldots p_n).
\]

(7)

\( M \) is a square matrix of dimension \((m - 1)n \times (m - 1)n\), and \( f \) is a column vector depending on the entrance pressure \( p_k = 1 \) \((k = 1, 2... n)\). The unknown node pressures can then be solved from Eq. (7), and the normalized surface impedance at the entrance of the tube network is obtained,

\[
\frac{1}{Z_{\text{eff}}} = \sum_{k=1}^{n} \Omega, \quad Z_k = -\frac{G p_k}{\rho c_0 \nu_k} = \frac{-GA p_k}{c_0 Q_k}.
\]

(8)
Here, $\Omega$ is the surface porosity of the tube network. The mass flow $Q_k$ can be obtained according to Eq. (4). The sound absorption coefficient is derived from the normalized surface impedance,

$$\alpha = \frac{4\text{Re}(Z_{\text{eff}})}{(1 + \text{Re}(Z_{\text{eff}}))^2 + (\text{Im}(Z_{\text{eff}}))^2}. \quad (9)$$

It is noted that the above method is also applicable if any tube in the network is removed. In addition, we add 0.8$R$ to the length of each tube to represent the thickness of chips.

GA in MATLAB is adopted to optimize the distribution of the tubes according to the targeted absorption spectrum. GA is an optimization algorithm that searches for the global optimal solution and does not need the derivative of the objective function (Goldberg and Holland, 1988). First, randomly generate the initial population of individuals, which are parents. Next, evaluate the fitness of each individual in that population. Then select the best-fit individuals for reproduction and generate new individuals through crossover and mutation operations in parents. If a fixed number of the generations are finished, then stop and return the optimized result. Otherwise, go to the next generation. During the optimization design of the network absorber, the objective function is the average sound absorption of the target frequency range. We use the default parameter settings of GA in MATLAB R2017b. For the optimization design of the network absorber in Fig. 3, the optimization variables form a $1 \times 117$ binary sequence. The population size is 200. The crossover probability is 0.8, and the mutation probability is 0.005. We emphasize here that the combination of the low reduced frequency model and GA is quite flexible to design tube networks for the programmed absorption spectrum. As demonstrated in Sec. III, it is hardly achieved with current absorption design strategy based on porous materials or perforated plate technique.

B. Experiments

We design and fabricate an optimized network absorber and experimentally measure its absorption performance.

The optimization target is complete absorption from 500 to 800 Hz. As shown in Fig. 3(a), the total thickness of absorber is assumed to be 82 mm ($\lambda/5$), of which 2 mm is the thickness of hard boundary. According to the precisions of experimental equipment and printing equipment, the area of incident surface is set to be $79.3 \times 79.3$ mm$^2$, and the radius of tubes is set as 1.5 mm. There are in total 20 tubes at the incident surface in a single layer of network; therefore, the distance between adjacent rows is 0.965 mm. The network absorber consists of 20 identical layers of tube network, of which the distance between adjacent layers is 0.965 mm. The network absorbers are fabricated by 3D printing (SR600SS, Uniontech, China) using DSM’s Somos® Imaging 8000 (DSM Functional Materials, Elgin, IL) as matrix (Young’s modulus: 2.2 GPa, density: 1200 kg/m$^3$). The measurements of the absorption coefficients are performed using the two-microphone transfer-function method [Bruel & Kjaer (Naerum, Denmark) type-4206T] according to ISO 10534–2 (1998) to obtain the sound absorption spectrums. We fabricate an impedance tube with a rectangle cross section ($79.7 \times 79.7$ mm$^2$) using aluminum alloy. During the experiments, the absorber is tightly stuck in the impedance tube, which is end up with a hard wall. The distance between two microphones is 5 cm. The cut-off frequency of the impedance tube is 2151 Hz.

III. RESULTS

The structure of the absorber consists of 20 identical layers of tube networks, and the results of absorption performance of the absorber are shown in Fig. 3. The red solid line represents the results predicted by the low reduced frequency model, and the blue dashed line represents the experimental results. The two lines agree well at resonant frequencies. The theoretical average absorption from 500 to 800 Hz is 86.8%, and measured average absorption from 500 to 800 Hz is 84.1%, realizing broadband and low-frequency sound absorption with a sub-wavelength thickness.

Afterward, we experimentally demonstrate the tunable network sound absorber. The absorber is decomposed into
seven parts, including three chips and four honeycomb plates, as shown in Fig. 4. To obtain different targeted sound absorption spectrums, we can change chips in the absorber. For example, we assemble a tunable network sound absorber [absorber 1 in Fig. 4(d)], whose network is the same as the “integrated” network shown in Fig. 3. The sound absorption performance of the tunable network is experimentally measured and represented by the yellow line in Fig. 4(a). There

FIG. 4. (Color online) (a) Comparison between integrated absorber and separated absorber. (b) Changing chips to increase the distance between the two peaks. (c) Changing chips to increase the absorption bandwidth. (d) Corresponding absorbers 1–5. (e) The separated absorber consists of honeycomb plates and chips.

FIG. 5. (Color online) Three low-frequency absorption targets are described by the gray zone (100% absorption from 500 to 1000 Hz), red zone (50% absorption from 500 to 1000 Hz), and blue zones (80% absorption from 650 to 950 Hz and 20% absorption from 1000 to 1300 Hz). Three optimized absorption results are described by the black solid line (network A), red dashed line (network B), and blue dashed-dotted line (network C).
exists some discrepancy of the sound absorption performance, which indicates that there are some small gaps between the honeycomb plates and chips. Overall, the tunable network works well at the designed frequency band, and its performance is consistent with its “integrated” version.

By changing the chips in the absorber, we assemble three different absorbers, and the corresponding sound absorption performances are measured, as shown in Fig. 4(b). For absorber 2 [Fig. 4(d)], the sound absorption peaks are located at 756 and 836 Hz. Compared with absorber 2, the two peaks of absorber 3 [Fig. 4(d)] with one different chip move away from each other (760 and 890 Hz). We further change one chip to form absorber 4 [Fig. 4(d)], for which the distance between two peaks becomes larger (650 and 870 Hz). We conclude that by changing chips, we can adjust the distance between two absorption peaks.

The absorber 4 shown in Fig. 4(d) is used for absorption from 500 to 900 Hz, and absorber 5 shown in Fig. 4(d) is used for absorption from 500 to 700 Hz. We experimentally obtain the sound absorption performance of the two absorbers, as shown in Fig. 4(c). It can be seen that the absorption peaks of the three absorbers (absorbers 1, 4, and 5) fall in the designed frequency band, respectively. Thus, we conclude that the working frequency range of the absorber can be adjusted by changing the chips, which are guided by the low reduced frequency model and GA.

We theoretically investigate the broadband sound absorption based on the GA optimization of the tube network absorber. Diverse target sound absorption spectrums can be realized at low frequencies. For example, as shown in Fig. 5, we design three networks according to three absorption spectrum targets, respectively. The first target absorption spectrum is assumed to be 100% absorption from 500 to 1000 Hz. We obtain network A and its sound absorption performance, which is represented by the black solid line in Fig. 5. Then we reduce the target to half absorption over the same frequency range. It is realized by network B, whose absorption performance is represented by the red dashed line. Both the absorption performances of network A and B agree well with the target absorption spectrums, respectively. The third target absorption spectrum is assumed to be 80% absorption from 650 to 950 Hz and 20% absorption from 1000 to 1300 Hz, represented by the blue zones. Its optimized network C realizes 80% absorption from 650 to 950 Hz and 20% absorption from 1000 and 1170 Hz, as shown in Fig. 5. The lengths of the three networks A–C in the propagating direction are 80 mm, and other parameters of the three networks, including radius $R$, distance between rows $L$, distance between layers $H$, number of rows $m$, and number of columns $n$, are listed in Table I.

Apart from tailored low-frequency absorption, we can also design networks to realize a wider absorption spectrum. We design another three networks corresponding to three different absorption targets, respectively. As shown in Fig. 6, the gray zone represents the target of 100% absorption from 300 to 3000 Hz, and the black solid line represents the absorption performance of the corresponding network D. We can see the network D can reach 97%–99% absorption from 700 Hz, represented by the black solid line. Then we decrease the absorption target to 80% or 70% over the same frequency range, and the corresponding optimized network E/F can reach an 80%/70% absorption target from 700 Hz. The three networks D–F are of the same length, 120 mm in the incident direction, and other geometric parameters are listed in Table I.

We find that the tube radius significantly influences the absorption performance. To address the sound absorption

### Table I. Geometric parameters of six optimized networks.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$ (mm)</td>
<td>0.3</td>
<td>0.2</td>
<td>0.7</td>
<td>0.2</td>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>$L$ (mm)</td>
<td>0.1</td>
<td>0.2</td>
<td>0.7</td>
<td>0.05</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>$H$ (mm)</td>
<td>0.1</td>
<td>0.3</td>
<td>0.75</td>
<td>0.05</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>$m$</td>
<td>6</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>$n$</td>
<td>9</td>
<td>11</td>
<td>11</td>
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performance of a network with a different radius, we choose a “full” network, where all tubes are turned on, and consider the average absorption coefficient from 300 to 4000 Hz as an indicator of absorption performance. We change the radius of tubes and distances $H$ and $L$. We set $m$ to 4, $n$ to 5, and thickness of network to 120 mm. We choose six cases for $H$ and $L$ and show how the absorption performances change with radius in Fig. 7(a). It can be seen that there exists one optimal radius corresponding to the best average absorption in every curve. The optimal radius decreases with the row distance $H$ (layer distance $L$) decreasing. When $H$ and $L$ are less than 0.1 mm, the optimal radius value is around 0.15 mm, which is approximately equal to the boundary layer thickness of the tube at low frequency range (Cai et al., 2018).

The above tube networks A–F are formed by identical layers, each of which is a two-dimensional (2D) network. For larger design space of optimization, we can add tubes at the nodes between two or more layers to optimize 3D networks. To compare with the one-layer network A, we design a two-layer network and a three-layer network to realize 100% absorption from 500 to 1000 Hz, which share the same geometric parameters. The networks and absorption results are all shown in Fig. 7(b). The red dotted line represents the results of the three-layer network, which is the best among the three networks. We conclude that 3D networks assembled with more different layers provide more paths for sound dissipation, having potentials for sound absorption.

IV. CONCLUSION

In this study, we propose and experimentally measure a tunable low-frequency sound absorber consisting of tube networks. The tube network can be optimized to provide broadband low-frequency absorption. We experimentally validate the optimized network, realizing 84.11% absorption from 500 to 800 Hz within an 82 mm thickness. By composing the network into honeycomb plates and chips, we can tune the absorption performance of the network absorber. Groups of chips are fabricated to carry out experiments on the tunable absorber. Furthermore, we found that there exists one optimal radius of tubes corresponding to best average absorption, and a 3D network can provide better absorption compared with a 2D network. The tunable network absorber has potential in new programmed absorbing material design.

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